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Multiple-Reservoir Scheduling Using β-Hill Climbing Algorithm

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Abstract: The multi-reservoir systems optimization problem requires defining a set of rules to recognize the water amount stored and released in accordance with the system constraints. Traditional methods are not suitable for complex multi-reservoir systems with high dimensionality. Recently, metaheuristic-based algorithms such as evolutionary algorithms and local search-based algorithms are successfully used to solve the multi-reservoir systems. β-hill climbing is a recent metaheuristic local search-based algorithm. In this paper, the multi-reservoir systems optimization problem is tackled using β-hill climbing. In order to validate the proposed method, four-reservoir systems used in the literature to evaluate the algorithm are utilized. A comparative evaluation is conducted to evaluate the proposed method against other methods found in the literature. The obtained results show the competitiveness of the proposed algorithm.

Keywords: β-Hill climbing, optimization, multi-reservior operation, partially and fully constrained, local search-based heuristic method.

1 Introduction

A water reservoir is a storage space of water where its operation relies on a sequence of rules to recognize the water amount stored and released according to the system constraints [9]. Operating multi-reservoirs is considered a hard decision-making activity which includes several decision variables and multi-objectives [26]. In the optimization context, the multi-reservoir operation is a complicated non-convex and non-linear large-scale optimization problem that is not easy to solve using the classical approaches [3]. The main objective when solving this problem is to seek the optimal combination of releases that leads to the maximum benefit generated throughout the multi-reservoir system. Earlier methods try to solve a simplified form of the problem [4].

Several techniques can be used to solve this problem such as linear programming (LP) [10, 29], nonlinear programming [11, 27, 31], and dynamic programming [15, 24]. These techniques are used to find the best schedule of the multi-reservoir systems. These traditional approaches are easy to use and efficient for restricted problems. However, they are not suitable for complex multi-reservoir systems with high dimensionality [9].

Recently, metaheuristic-based approaches have been introduced to tackle multi-reservoir operation problems (MROPs). Metaheuristic is a general optimization framework that can be applicable to solve a wide variety of optimization problems. Normally, it is initiated with a set of random solution(s) and iteratively

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refine these solution(s) based on built-in learning operators that are controlled by probability parameters. These algorithms are able to intelligently navigate the problem search space through exploiting the accumulative search (i.e. exploitation) and exploring the not-yet-visited regions (i.e. exploration). Metaheuristic-based approaches can be categorized into three classes: evolutionary-based algorithms (EA), swarm-based algorithms (SA), and trajectory-based algorithms (TA) [12, 14].

EA is initiated with a population of randomly constructed individuals. These individuals are improved generation after generation using recombination, mutation, and selection operators to come up with the fittest population. This technique is very powerful in exploring several search space regions simultaneously. However, it is weak to exploit the search space region to which it converges and finds a precise local optimal solution. Several EAs have been introduced for MROPs such as genetic algorithm (GA) [20, 30], and harmony search algorithm (HSA) [5, 16, 17, 22, 23].

SA is a new trend in optimization where the behavior of the swarm searching for food to optimize their living is imitated. This is done by local interaction between the swarm members which cannot be achieved by a lone-acting component in the swarm system. The SAs used to tackle MROPs are ant colony optimization [21, 26], honey bee mating optimization [18], particle swarm optimization [25], bat algorithm [6, 13], water cycle algorithm [19], and weed optimization algorithm [9].

A TA, which is the main interest of this paper, is initiated by a randomly generated solution that is guided through the search process. Iteration by iteration, that solution undergoes changes using a learning mechanism normally based on the neighborhood navigation until a solution is obtained. The final solution is called local optimal solution in which it cannot be further improved. TAs used for MROPs include simulated annealing [28].

Recently, a new trajectory-based method called β-hill climbing has been proposed by Al-Betar for optimization problems [7]. It initiated with a single stochastic solution. Iteratively, this solution undergoes changes using two operators: $\mathcal{N}$-operator used for neighborhood navigation and β-operator used for random navigation. This iterative process keeps going until an optimal solution is achieved. β-hill climbing is simple and easy to tailor to any optimization problem, and its structure is very promising. Therefore, it is tailored to several optimization problems such as feature selection [2], text clustering [1], and sudoku [8].

In this paper, the β-hill climbing algorithm is used to model the MROPs. As a MROP is a multidimensional, non-convex, and non-linear constrained problem, the feasibility of the solution is preserved during the search through a repair process. The proposed method is evaluated using de facto four-reservoir system used to evaluate the performance of the previous methods. The results reveal that the proposed method is capable of yielding a viable solution for this problem similar to or better than that obtained by the other comparative methods.

The remaining part of this paper is organized as follows: the second section provides definitions and mathematical formulations for the MROPs. The third section presents the proposed β-hill climbing algorithm for MROP. The fourth section shows the experiments and comparative results of the proposed method against others using four-reservoir systems. The final section concludes the paper and suggests some directions for future investigations.

2 Multiple-Reservoir Operation Problem: Definition and Formulation

In this section, the MROP will be formulated and described. After that, a dataset of four-reservoir systems is used to show the implementation of the mathematical formulation using scalable MROP.

2.1 General Definition

The multiple-reservoir operation problem is a complex, non-linear, non-convex and multi-objective optimization problem which cannot be easily handled by the classical approaches. Due to its nature, heuristic techniques usually provide better performance to solve the problem.

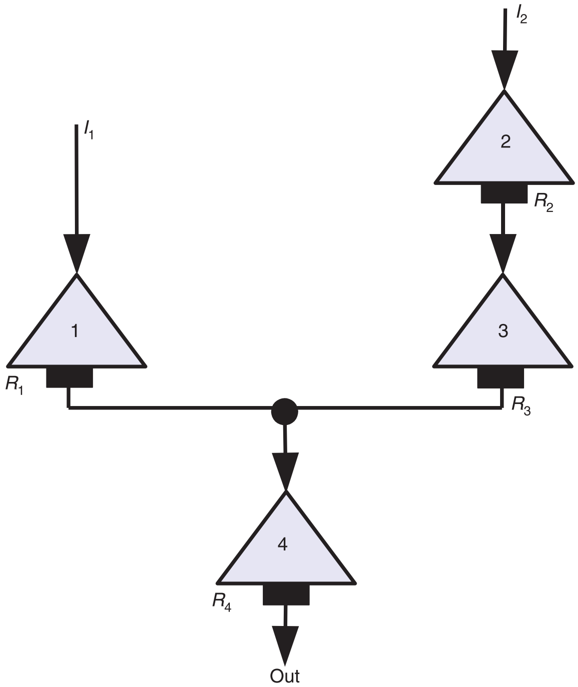

Figure 1: Four Reservoir System.

Initially, across the river, there are multi-reservoirs connected together either sequentially or in parallel in order to generate a hydro-power or to be used for irrigation. Hydro-power generation can be accomplished through each reservoir, and the discharges outflow through the turbines. For an example of a four-reservoir system, see Figure 1. In this example, reservoir #4 is just used for irrigation.

2.2 Mathematical Formulation

The objective function is a linear function with a single objective that is formulated to maximize the benefits of hydro-power generation and irrigation over a specific period. It can be defined as follows [16]:

$$\max Z(\mathbf{R}) = \sum_{i=1}^{K}\sum_{t=1}^{T} b_i(t)\times R_i(t) + \sum_{i=1}^{K}\sum_{t=1}^{T} p_i(t)\times R_i(t) \tag{1}$$

where $R_i(t)$ is a discrete water release in the time t from the reservoir i. The $p_i(t)$ is the benefit obtained from the hydro-power generation and the $b_i(t)$ is the benefit obtained from irrigation. The water release $R_i(t)$ should be located between lower and upper limits as follows:

$$R^{\text{MIN}}(t) \le R(t) \le R^{\text{MAX}}(t)$$

The model should satisfy the continuity constraint as follows:

$$\mathbf{S}_i(t+1) = \mathbf{S}_i(t) + \mathbf{I}_i(t) + \mathbf{M}\cdot\mathbf{R}_i(t)$$

where $\mathbf{S}_i(t)$ = vector of reservoir storages, $\mathbf{I}_i(t)$ = vector of inflows to each reservoir, and $\mathbf{M}$ = reservoir connection matrix.

The reservoir storage amount $S_i(t)$ should have a value between the lower and upper bounds.

$$S^{\text{MIN}}(t) \le S(t) \le S^{\text{MAX}}(t)$$

2.3 Four-Reservoir System Definition and Formulation

A four-reservoir system is used to illustrate the multi-reservoir operational problem. Four reservoirs are connected together sequentially as shown in Figure 1. As can be noted, reservoir #4 has only outflow connection which is just used for irrigation since the discharge is connected directly to the farms.

The system has only four reservoirs; the connectivity matrix **M** of size 4×4 is shown as follows:

$$\mathbf{M}=\begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 1 & -1 & 0\\ 1 & 0 & 1 & -1\end{bmatrix}. \tag{2}$$

$M_{i,j}$ can take one of three values {–1, 0, 1}. The value of $M_{i,j}=1$ when there is a connection between the reservoir i and the reservoir j. On the other hand, the value of $M_{i,j}=0$ indicates that there is no connection between the two reservoirs.The value $M_{i,j}=-1$ indicates that the connection cannot be made between the two reservoirs.

In the four-reservoir system the benefits from irrigation of reservoir #4 (i.e. b_4) and the hydro-power generation from all the reservoirs [p_i where $i\in(1\ldots,4)$] is presented in Table 1. This four-reservoir system example is taken from [30], and it represents 12 2-h operating periods.

As shown in Figure 1, the solution of the four-reservoir system is formulated as a vector of releases $\mathbf{R}=(R_1, R_2, R_3, R_4)$ from the reservoir to the turbines where the feasible range of each reservoir release is given as follows:

$$0.0\le R_1\le 3 \tag{3}$$

$$0.0\le R_2,\ R_3\le 4 \tag{4}$$

$$0.0\le R_4\le 7 \tag{5}$$

The continuity constraints is satisfied based on the feasible range of the reservoir storage schedule which is given for a four-reservoir system as follows:

$$0.0\le S_1,\ S_2,\ S_3\le 10 \tag{6}$$

$$0.0\le S_4\le 15 \tag{7}$$

Note that in operation period #0, the reservoir storages are modelled using the following equality constraints:

Table 1: Rates of Benefit Parameters b_i.

t	$b_1(t)$	$b_2(t)$	$b_3(t)$	$b_4(t)$	$b_5(t)$
0	1.1	1.4	1.0	1.0	1.6
1	1.0	1.1	1.0	1.2	1.7
2	1.0	1.0	1.2	1.8	1.8
3	1.2	1.0	1.8	2.5	1.9
4	1.8	1.2	2.5	2.2	2.0
5	2.5	1.8	2.2	2.0	2.0
6	2.2	2.5	2.0	1.8	2.0
7	2.0	2.2	1.8	2.2	1.9
8	1.8	2.0	2.2	1.8	1.8
9	2.2	1.8	1.8	1.4	1.7
10	1.8	2.2	1.4	1.1	1.6
11	1.4	1.8	1.1	1.0	1.5

$$S_1(0),\ S_2(0),\ S_3(0),\ S_4(0) = 5 \tag{8}$$

while the reservoir storages in operation period #12 are given as the following equality constraints:

$$S_1(12),\ S_2(12),\ S_3(12) = 5,\quad S_4(12) = 7 \tag{9}$$

The above equality constraints must be satisfied to ensure that the system schedule has no conflicts. The inflow for reservoirs 1 and 2 has fixed values during the system operation as follows:

$$I_1 = 2,\ I_2 = 3 \tag{10}$$

3 Scheduling Multiple-Reservoir System Using β-Hill Climbing

In this section, the β-hill climbing optimizer is tailored to solve the MROP. As any other local search-based method, β-hill climbing is initiated by a single randomly generated solution of releases $\mathbf{R} = (R_1, \ldots, R_n)$ where n refers to the total number of reservoirs. The value range for each release is given in advance as shown in the previous section. The initial solution is generated in accordance with operation period constraints. Note that the value range for each reservoir release is taken from a continuous and feasible domain.

The initial solution is evaluated using the objective function formulated in equation (1). The values of irrigations and hydro-power benefits are given in advance and are used to assess the quality of the generated solution. In the β-hill climbing optimizer, the initial solution undergoes several changes iteration by iteration until the optimal solution is reached. During each iteration, three operators are performed, hopefully ensuring desirable changes: $\mathcal{N}$-operator, β-operator, and selection operator.

In $\mathcal{N}$-operator, a release R_i, $i \in [1, 2, \ldots, n]$, for the solution is randomly selected then the value of the selected release is updated as follows:

$$R_i' = R_i \mp U(0, 1) \times \mathcal{N} \quad \exists i \in [1, n]$$

The parameter $\mathcal{N}$ is the distance between the current solution and the neighboring solution. Notably, the reservoir releases and the storage constraints must be fulfilled when any value is selected. Thus, the system will neglect any value that leads to infeasibility. If the feasibility is not met, a simple repair process which perseveres the neighboring solution is invoked. This operator utilizes the idea of making use of the accumulative search where the neighboring solution is a small change to the current solution. Al-Betar [7] considered this operator as the main source of exploitation.

In β-operator, all releases in the solution are checked for whether or not these are to be changed from the possible range of the release values based on the value of the β parameter as follows:

$$R_i'' \leftarrow \begin{cases} R_k & r \le \beta \\ R_i' & \text{otherwise.} \end{cases}$$

where β is a probability that ranges between 0 and 1. The r operator generates a uniform digit from the range 0 and 1. The R_k is selected randomly from the feasible range of the reservoir release R_i. Note that the feasible range is the one that satisfies both the release and the storage constraints. Apparently, this operator either keeps the value of release reservoir R_i' unchanged or updates it from the possible value range. Therefore, this operator works similarly to the uniform mutation operator in GA which is the main source of exploration.

Finally, the selection operator performs a greedy selection where it will replace the new release solution $\mathbf{R}''$ with current one if and only if the objective function value of the new release solution $Z(\mathbf{R}'')$ is better than the current one $Z(\mathbf{R})$. The algorithm steps are presented in the pseudo-code shown in Algorithm 1.

Algorithm 1: β-Hill Climbing Pseudo-Code for Multiple-Reservoir System.

$R_i = R_i^{Min} + (R_i^{Max} - R_i^{Min}) \times U(0, 1)$, $\forall i = 1, 2, 3$, and 4 {The initial solution R}
Calculate $Z(R)$ {$Z(R)$ is the objective function calculated by Eq. (1)}
$itr = 0$
while ($itr \leq$ Max_Itr) **do**
  $R' = R$
  $k \in [1, 4]$
  $R'_k = R_k \pm U(0, 1) \times \mathcal{N}$.
  $R'' = R'$
  for $i = 1, \ldots, N$ **do**
    if ($r \leq \beta$) **then**
      $R''_i = R_i^{Min} + (R_i^{Max} - R_i^{Min}) \times U(0, 1)$
    end if {$r \in [0, 1]$}
  end for
  Calculate $Z(R'')$
  if $Z(R'') \leq Z(R)$ **then**
    $R = R''$
    $Z(R) = Z(R'')$
  end if
  $itr = itr + 1$
end while

Table 2: Performance of the Proposed β-Hill Climbing Algorithm with Different β Values for Four-Reservoir System.

	Max	Min	Avg	SD	Success rate
$\beta = 0$	392.10	389.00	390.93	1.11	100%
$\beta = 0.001$	**395.60**	388.90	391.70	2.11	100%
$\beta = 0.010$	390.50	384.20	388.17	2.00	90%
$\beta = 0.100$	375.40	368.20	372.34	2.18	80%
$\beta = 0.300$	366.10	350.00	363.33	4.96	60%

Value in bold refers to the best benefit obtained.

Table 3: Optimal Water Release Schedule of the Best Solution Obtained by the Proposed Method.

Time	Dam 1	Dam 2	Dam 3	Dam 4
0	1.0	4.0	0.0	1.0
1	1.0	2.0	2.0	0.0
2	0.0	2.0	3.0	7.0
3	1.0	2.0	4.0	7.0
4	3.0	4.0	4.0	7.0
5	3.0	3.0	4.0	7.0
6	3.0	4.0	4.0	7.0
7	3.0	3.0	3.0	7.0
8	2.0	4.0	4.0	7.0
9	3.0	4.0	4.0	7.0
10	3.0	4.0	3.0	1.0
11	1.0	0.0	1.0	0.0

4 Results and Discussion

In this section, the proposed β-hill climbing algorithm is evaluated using a four-reservoir system to illustrate its efficiency and effectiveness. The values of the four-reservoir system parameters are extracted from [30]. The β parameter used in the proposed algorithm is studied with five different values as reported in Table 2. These

Table 4: Explanation of How Optimal Water Release Schedule Obtained in Table 3 Affects the Multi-Reservoir System.

Time	$I_i(t)$	$S_i(t)$	$R_i(t)$	$S_i(t+1)$
0				
Dam 1	2	5	1	6
Dam 2	3	5	4	4
Dam 3	4	5	0	9
Dam 4	1	5	1	5
1				
Dam 1	2	6	1	7
Dam 2	3	4	2	5
Dam 3	2	9	2	9
Dam 4	3	5	0	8
2				
Dam 1	2	7	0	9
Dam 2	3	5	2	6
Dam 3	2	9	3	8
Dam 4	3	8	7	4
3				
Dam 1	2	9	1	10
Dam 2	3	6	2	7
Dam 3	2	8	4	6
Dam 4	5	4	7	2
4				
Dam 1	2	10	3	9
Dam 2	3	7	4	6
Dam 3	4	6	4	6
Dam 4	7	2	7	2
5				
Dam 1	2	9	3	8
Dam 2	3	6	3	6
Dam 3	3	6	4	5
Dam 4	7	2	7	2
6				
Dam 1	2	8	3	7
Dam 2	3	6	4	5
Dam 3	4	5	4	5
Dam 4	7	2	7	2
7				
Dam 1	2	7	3	6
Dam 2	3	5	3	5
Dam 3	3	5	3	5
Dam 4	6	2	7	1
8				
Dam 1	2	6	2	6
Dam 2	3	5	4	4
Dam 3	4	5	4	5
Dam 4	6	1	7	0
9				
Dam 1	2	6	3	5
Dam 2	3	4	4	3
Dam 3	4	5	4	5
Dam 4	7	0	7	0
10				
Dam 1	2	5	3	4
Dam 2	3	3	4	2
Dam 3	4	5	3	6
Dam 4	6	0	1	5

Table 4 (continued)

Time	$I_i(t)$	$S_i(t)$	$R_i(t)$	$S_i(t+1)$
11				
Dam 1	2	4	1	5
Dam 2	3	2	0	5
Dam 3	0	6	1	5
Dam 4	2	5	0	7

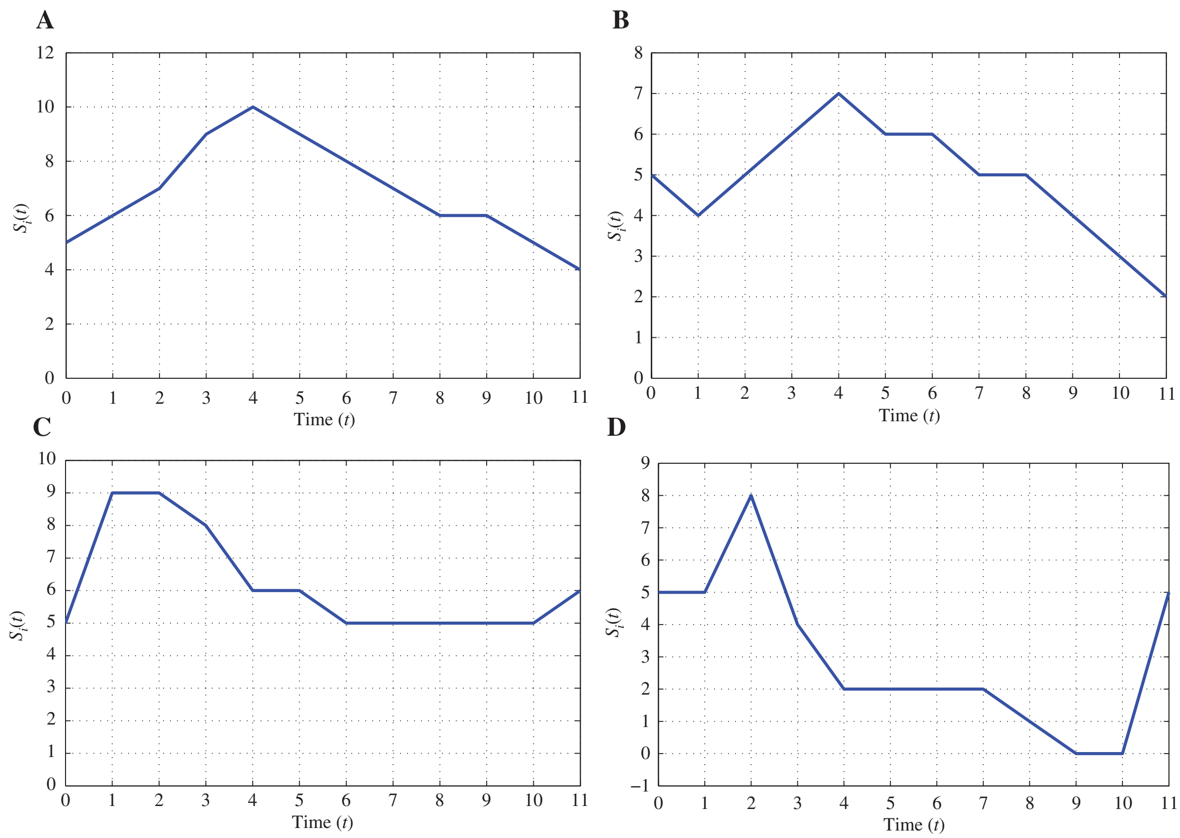

Figure 2: The Changing in the Storage $\mathbf{S}_i(t)$ for Each Dam over the Time Interval.
(A) Dam 1, (B) Dam 2, (C) Dam 3 and (D) Dam 4.

values are carefully selected and experimented based on the recommendation of the original β-hill climbing algorithm [7] and based on an initial experiment exhaustively done. It is worthy to note that the higher value of the β parameter leads to higher rate of exploration. In this table, the maximum (Max), minimum (Min), and average (Avg) benefits as well as the standard derivation (SD) of the solutions obtained by the proposed algorithm over 10 runs are recorded. In addition, the rate of success to reach feasible solutions over 10 runs is also reported in the same table. The maximum number of iterations used is fixed to 10,000. The best benefit obtained (the highest is best) is highlighted using bold font in the table.

Apparently, the proposed β-hill climbing algorithm with β parameter fixed to 0.001 obtained the best solution with the benefit (395.60), whereas the proposed method with the higher rate of β parameter leads to higher rate of undesirable exploration. Table 3 illustrates the optimal water release schedules for the best solution obtained. Note that in the table, the numbers refer to the water release schedule of each dam at a specific time. For example, at time 0, dam 1 will release the water of amount 1 unit; dam 2 will release the amount of 4, while dam 3 does not release any amount and dam 4 releases 1 water release unit.

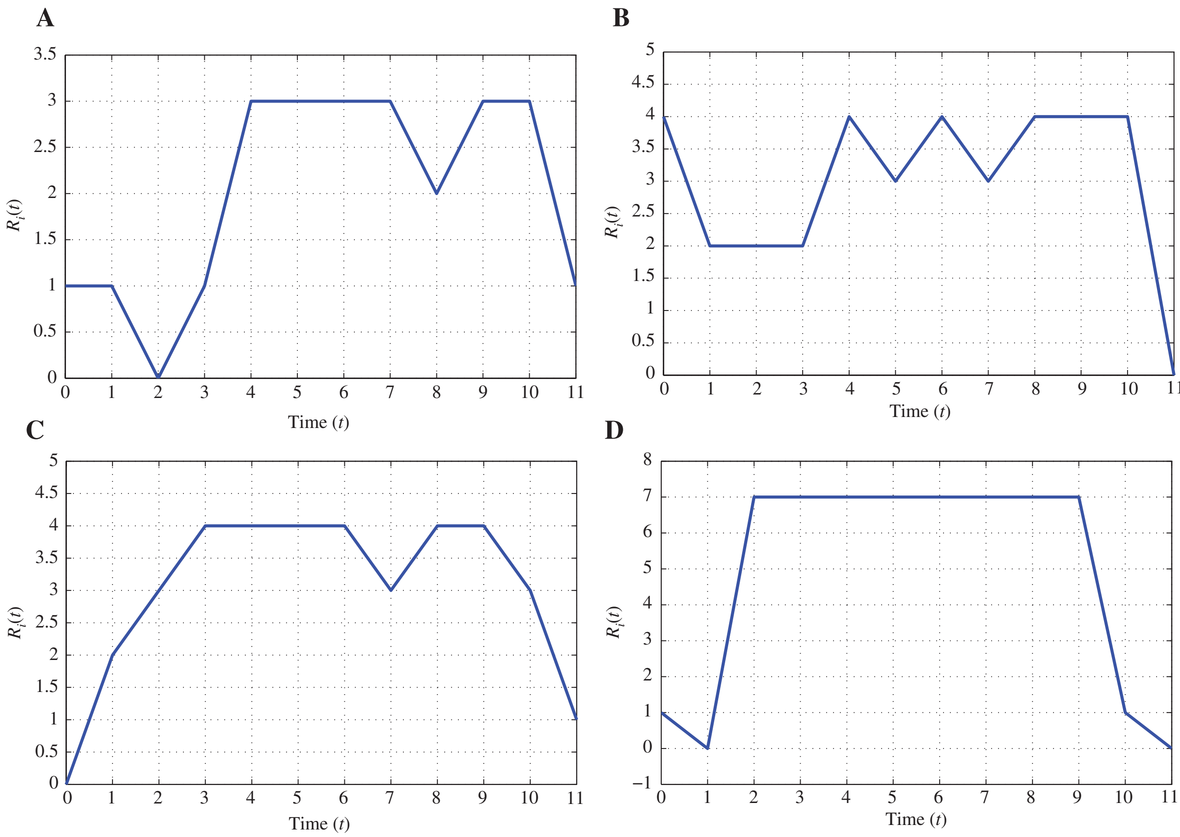

Figure 3: The Changing in the Release $\mathbf{R}_i(t)$ for Each Dam over the Time Interval. (A) Dam 1, (B) Dam 2, (C) Dam 3 and (D) Dam 4.

Table 5: Comparison of Optimal Water Release Schedules for Dam 2.

Time	LP	HS	βHC
0	4	4	4
1	1	1	2
2	2	2	4
3	0	0	0
4	3	3	1
5	4	4	4
6	4	4	4
7	4	4	4
8	4	4	4
9	2	2	3
10	4	4	4
11	4	4	2

Table 4 shows how the optimal water release schedule obtained by β-hill climbing algorithm shown in Table 3 affects the whole system at each time t. In Table 4, the $I_i(t)$, $S_i(t)$ and $R_i(t)$ for each dam is used to calculate the storage of the next time $S_i(t+1)$. Notably, the optimal solution preserves the feasibility of the system constraints at each time t. The changes of the storage and of the water release against the time t for each reservoir are drawn in Figures 2 and 3 consecutively.

It can be noted from the optimal water release schedule shown in Table 3 and its values explained in Table 4 that the storage $S_i(t)$ at any time t provides a good impression about the hydropower within

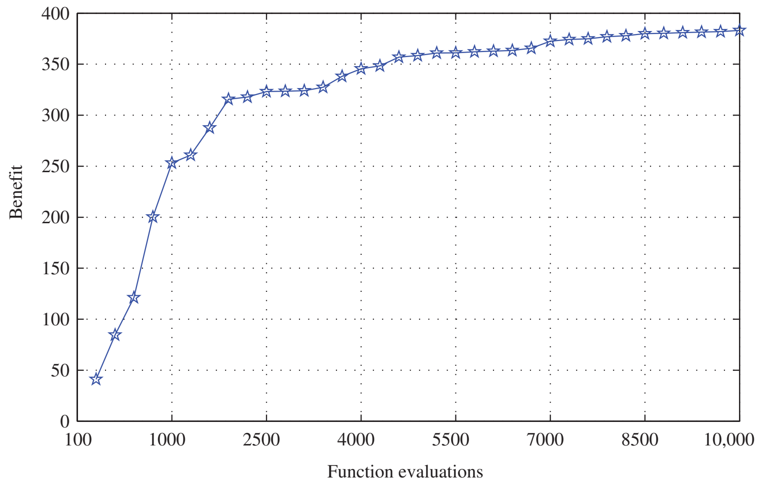

Figure 4: Convergence Behavior of β-Hill Climbing Algorithm.

this schedule. This is because within the operation time interval, there are water amount storages in the reservoirs/dams (1, 2 and 3). Accordingly, the process of water releasing through these reservoirs for the purpose of hydropower did not affect the water storage of these reservoirs which will perfectly manage the release process. For reservoir 4, it is noted that some storage values at some times are 0. However, these cases do not affect the release process for irrigation. In the last time ($t=0$), the optimal water release schedule is able to return the hydropower/irrigation system to the initial storage for each reservoir [$S_i(0)=5, \forall i=(1, \ldots, 4)$]. In general, the main reason of establishing the reservoirs is to take a benefit from the water storage in order to produce the hydropower as well as to make use of the stored water in the irrigation process.

The results of β-hill climbing algorithm are compared with the LP and HSA for dam 2 as illustrated in Table 5. As shown in this table, the proposed β-hill climbing algorithm obtains the same result as the LP and the HS in 7 out of 12 times. Figure 4 plots the best benefits at each iteration of the proposed β-hill climbing algorithm when exploring the search space. The curve line in this plot shows the correlation between the number of evaluations and the best benefit. Clearly, the benefit of the solution is enhanced till the last iteration of the search.

5 Conclusion and Future Research

In this paper, a MROP which is a multi-objective complex optimization problem has been tackled by a recent optimization local search-based metaheuristic algorithm called β-hill climbing. The MROP is concerned with seeking the optimal scheduling of the reservoir releases that leads to the maximum benefit power generation and irrigation throughout the multi-reservoir system. The solution is represented as a vector of releases, and the optimality is achieved according with some equality and inequality constraints related to the periodic reservoir storage. β-hill climbing begins with a provisional solution. This solution is iteratively updated based on three operators: $\mathcal{N}$-operator which is responsible for exploitation, β-operator which is responsible for exploration, and finally the selection operator which behaves like greedy search.

The proposed method is evaluated against a de facto test system of a four-reservoir operation problem. This problem has four reservoirs connected together sequentially. Interestingly, the proposed method is able to solve the four-reservoir problem efficiently and to produce a solution that is close to the optimal. In order to validate the performance of the proposed algorithm against β parameter, different values of the β parameter are experimented with. As a conclusion, when the value of the β parameter is close to 0, the performance of the proposed method becomes better.

This initial investigation of applying the β-hill climbing to the MROP shows viable performance. In the future, other kinds of MROP can be used. Also, other factors and features of the MROP can be included in the representation of the problem to show more valuable results.

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