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Analytic solutions for a modified fractional three wave interaction equations with conformable derivative by unified method

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Abstract The present work implements the unified method to a class of fractional partial differential systems corresponding to modified fractional three wave interaction equations (FTWIEs). The fractional derivative of the three waves envelopes is considered under the conformable sense. The conformable FTWIEs is derived in the (3 + 1) dimensions under the conformable time-fractional derivative of order $\alpha \in (0, 1]$ based on a modification of the Lax-pair created by Zakharov-Manakov which includes three-dimensional velocity vector dotted with the usual del operator. Then, the unified method for creating solutions for nonlinear evolution FTWIEs is applied. Subsequently, a systematic algorithm is used to obtain an infinite set of exact rational solutions to the novel constructed system. By randomly selecting the special values for the parameters, three-dimensional graphs are also given for different patterns. The obtained solutions might play an essential role in many other nonlinear evolution equations that occur in the fields of engineering and mathematical physics.

1. Introduction

The concept of fractional derivative has been introduced during the past decades, it generalized the classical definition of the derivative of integer order to a derivative of fractional order. It provides an attractive mechanism to explain memory and hereditary characteristics of complex systems, including fluid flow, electrodynamics, quantum mechanics, rheology, damping laws, thermoelectricity, and diffusion processes [1–6]. Several physical and engineering problems can be typically modeled by partial differential equations of the fractional order due to its accuracy in providing and neglecting the influence of external forces [7–12]. Therefore, an urgent need arose to obtain exact or approximate solutions or merely numerical

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ones to these problems. Actually, there are many definitions of fractional derivatives in literature such as the definitions given by Caputo, Riemann–Liouville, Caputo-Fabrizio, Atangana-Baleanu, Grünwald-Letnikov and many more [13–17].

A Lax-pair is a set of two time dependent operators when constructed carefully gives a nonlinear evolution system of partial differential equations which represents a model of an interesting mathematical problem. A well known example of Lax-pair set of operators is the pair which gives the nonlinear evolution of three wave interaction equations. Anyhow, the idea of Peter-Lax that was proposed in 1968 of constructing a set of couples of two operators satisfying some conditions leads to many interesting integrable systems of nonlinear evolution equations, including the Korteweg-de Vries equation [18], sine Gorden equation [19], discrete Boussinesq equation [20], and so on. Among these models is the system of nonlinear evolution of the three wave interaction equations [21–24], which has many applications in applied mathematics, plasma, fluid mechanics, wave fields, and propagating of dispersive nonlinear waves. Indeed, such system is a mathematical model for three interacting nonlinear soliton waves with ordered velocities, when reaching the interaction zone an energy transfer can occur causing many important consequences and physical applications. Under some conditions they also represent the interaction of three propagating optics wave packets in quadratic nonlinear media. However, many numerical schemes and analytical methods were used and developed to study the nature of the interaction between these nonlinear waves. The basic motivation in studying the three wave interaction equations is to get a soliton solution or rational solution and to investigate solutions behavior in the direction of time, therefore many successful efforts have been achieved in literature and some of them are mentioned in the references [22–24].

The classical model of three wave interaction equations has been suggested under the following assumptions: Let L and M be two operators, called Lax-pair of the three wave interaction equations, defined by Zakharov and Manakov [21–23] such that

$$\begin{array}{lll} L & = & \mathbb{i} A_{3\times3}\frac{\partial}{\partial z} + [A_{3\times3}, Q_{3\times3}], \\ M & = & \mathbb{i} B_{3\times3}\frac{\partial}{\partial z} + [B_{3\times3}, Q_{3\times3}], \end{array} \tag{1}$$

where $[X, Y] = XY - YX, \mathbb{i} = \sqrt{-1}, A$ and B are two real-valued matrices whose elements are non-zero real numbers chosen so that $a_{ij} = a_i\delta_{ij}, b_{ij} = b_i\delta_{ij}, a_3 < a_2 < a_1$, while the matrix Q is given by

$$Q(z,t) = \begin{pmatrix} 0 & \alpha_1 u_2(z,t) & \alpha_2 u_1(z,t) \\ -\alpha_1 u_2^*(z,t) & 0 & \alpha_3 u_3(z,t) \\ -\alpha_2 u_1^*(z,t) & -\alpha_3 u_3^*(z,t) & 0 \end{pmatrix},$$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \frac{1}{\det(R)} I_{3\times3} \begin{pmatrix} \gamma_1\sqrt{(a_1-a_3)(a_2-a_3)} \\ \gamma_2\sqrt{(a_1-a_2)(a_2-a_3)} \\ \gamma_3\sqrt{(a_1-a_2)(a_1-a_3)} \end{pmatrix}, \tag{2}$$

$$R = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 1 & 1 & 1 \end{pmatrix}, \qquad \gamma_1^2 = \gamma_2^2 = \gamma_3^2 = 1,$$

in which $*$ denotes the complex conjugate, $I_{3\times3}$ is the identity matrix of order $3\times3, u_i(z,t)$ is the packet of the i^{th} wave propagates at the z axis with time t. Then, the three wave interaction equations can be produced in the following matrix form

$$W = \frac{\partial L}{\partial t} = \left[A, \frac{\partial Q}{\partial t}\right] = \mathbb{i}[L, M], \tag{3}$$

provided that $w_{ii} = 0$ and $w_{ij} = w_{ji}^*$. While the lower diagonal elements of W are given by

$$\begin{array}{lll} \frac{\partial u_1}{\partial t} + v_1\frac{\partial u_1}{\partial z} & = & \mathbb{i}\gamma_1 u_2 u_3, \\ \frac{\partial u_2}{\partial t} + v_2\frac{\partial u_2}{\partial z} & = & \mathbb{i}\gamma_2 u_1 u_3^*, \\ \frac{\partial u_3}{\partial t} + v_3\frac{\partial u_3}{\partial z} & = & \mathbb{i}\gamma_3 u_1 u_2^*, \end{array} \tag{4}$$

where $v_1 = \frac{a_1b_3 - a_3b_1}{a_3 - a_1}, v_2 = \frac{a_1b_2 - a_2b_1}{a_2 - a_1}$, and $v_3 = \frac{a_2b_3 - a_3b_2}{a_3 - a_2}$.

Consequently, the mathematical model (4) is called the nonlinear evolution of three wave interaction system, which represents the interaction of three propagating waves in the z axis so that the i^{th} wave packet $u_i(z,t)$ propagates with velocity v_i [25]. There are many methods used effectively to get exact solutions for this model. For example, the inverse scattering transform method [21], Darboux dressing transformation method [26], $\overline{\delta}$-dressing method [27], spectral method [22], tanh–coth method [28], extended tanh method [29,30], multiple scale perturbation technique, and linear superposition principle [31].

The basic motivation of the current study is to do a simple modification on the Lax-pair operators to include the three-dimensional velocity vector dotted with the usual del operator, where the conformable fractional time derivative is used instead of the usual first-order partial derivative of the envelopes of the three waves with respect to time. The result is a new similar system of three partial differential equations which is considered more general than the classic original system. Subsequently, the application of the unified method (UM) is extended to obtain solutions for nonlinear evolution equations utilizing an accurate algorithm that gives an infinite set of exact rational solutions of the generalized system.

The order of this study goes as follows: Section 2 recalls some basic concepts of the conformable fractional derivative. Section 3 contains the derivation of three wave interaction equations under the conformable time-fractional derivative. The solution methodology used for handling the generalized proposed problem is presented in Section 4. In section 5, we apply the unified method and state the steps lead to obtain the rational solutions. While some applications together with 3D graphs of solution behavior are given in Section 6.Finally, a brief conclusion is outlined in Section 7.

2. Conformable fractional derivative

In this section, we present some definitions and properties of fractional calculus and investigate the description of the conformable fractional derivative that will be used throughout the rest of this study. It is introduced in the sense of limits to cover the deficiencies of the other existing concepts [32–40], and given in the following definition:

Definition 2.1 [32]: If $f(t) : \mathbb{R}^+ \longrightarrow \mathbb{R}$, then the conformable fractional derivative of order $\alpha \in (0,1]$ is defined as

$$D_t^\alpha f(t) = \underset{\epsilon \longrightarrow 0}{Lim} \frac{f\left(t + \epsilon t^{1-\alpha}\right) - f(t)}{\epsilon}, t > 0. \tag{5}$$

The following theorem covers some important properties of conformable fractional derivative:

Theorem 2.1 [33]: If $f(t)$ and $g(t)$ have conformable fractional derivatives for all $t > 0$, and m_1 and m_2 are real constants, then we have the following:(i) $D_t^\alpha(m_1 f(t) + m_2 g(t)) =$

$m_1 D_t^\alpha(f(t)) + m_2 D_t^\alpha(g(t))$.(ii) $D_t^\alpha(.)$ satisfies the classical quotient and multiplication rules so that

$D_t^\alpha\left(\frac{f(t)}{g(t)}\right) = \frac{g(t)D_t^\alpha(f(t)) - f(t)D_t^\alpha(g(t))}{(g(t))^2}$,

$D_t^\alpha(f(t)g(t)) = f(t)D_t^\alpha(g(t)) + g(t)D_t^\alpha(f(t))$.

(iii) $D_t^\alpha(.)$ operates on t^β similarly as the classical derivative does when $\alpha = 1$ so that

$D_t^\alpha(t^\beta) = \beta t^{\beta-\alpha}, \forall \beta \in \mathbb{R}$.

(iv) $D_t^\alpha(.)$ operates on the composition of two functions according the following relation:

$D_t^\alpha((f \circ g)(t)) = t^{1-\alpha} g'(t) f'(g(t))$.

Corollary 2.1: If we consider $g(t) = t$, then $D_t^\alpha(f(t)) = t^{1-\alpha} f'(t)$.

3. Formulation of Conformable FTWIEs

In this section, the fractional three wave interaction equations (FTWIEs) is derived in the (3 + 1) dimensions under the conformable time-fractional derivative of order $\alpha \in (0,1]$ based on some modifications of Zakharov-Manakov concept of Lax-pair operators.

Definition 3.1: Consider the following operators:

$$\begin{aligned} L &= \mathbb{I} A_{3\times3} \overrightarrow{v} \odot \overrightarrow{\nabla} + [A_{3\times3}, Q_{3\times3}], \\ M &= \mathbb{I} B_{3\times3} \overrightarrow{v} \odot \overrightarrow{\nabla} + [B_{3\times3}, Q_{3\times3}], \end{aligned} \tag{6}$$

where $\odot$ is the dot product, $[X,Y] = XY - YX, \mathbb{I} = \sqrt{-1}, A$ and B are 3×3 matrices whose elements are non-zero real numbers chosen so that $a_{ij} = a_i\delta_{ij}, b_{ij} = b_i\delta_{ij}, a_3 < a_2 < a_1$, $\overrightarrow{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ and $\overrightarrow{\nabla}(.) = \frac{\partial(.)}{\partial x}\hat{i} + \frac{\partial(.)}{\partial y}\hat{j} + \frac{\partial(.)}{\partial z}\hat{k}$, while the elements of the matrix Q are the i^{th} wave $u_i\left(\overrightarrow{X}, t\right)$ and given by

$$Q\left(\overrightarrow{X}, t\right) = \begin{pmatrix} 0 & \alpha_1 \mathbb{I} u_2\left(\overrightarrow{X}, t\right) & \alpha_2 u_1\left(\overrightarrow{X}, t\right) \\ -\alpha_1 \mathbb{I} u_2^*\left(\overrightarrow{X}, t\right) & 0 & -\alpha_3 \mathbb{I} u_3\left(\overrightarrow{X}, t\right) \\ -\alpha_2 u_1^*\left(\overrightarrow{\mathbf{X}}, \mathbf{t}\right) & \alpha_3 \mathbb{I} \mathbf{u}_3^*\left(\overrightarrow{\mathbf{X}}, \mathbf{t}\right) & \mathbf{0} \end{pmatrix}, \tag{7}$$

in which

$$\overrightarrow{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3, \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \frac{1}{\det(R)} \begin{pmatrix} \sqrt{\gamma_1\gamma_3}\sqrt{(a_1 - a_3)(a_2 - a_3)} \\ \sqrt{\gamma_2\gamma_3}\sqrt{(a_1 - a_2)(a_2 - a_3)} \\ \sqrt{\gamma_1\gamma_2}\sqrt{(a_1 - a_2)(a_1 - a_3)} \end{pmatrix},$$

$$R = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 1 & 1 & 1 \end{pmatrix}, \text{ and } \quad \gamma_1^2 = \gamma_2^2 = \gamma_3^2 = 1.$$

Consequently, the corresponding FTWIEs can be constructed from the following 3×3 matrix representation

$W = D_t^\alpha L = \left[A, D_t^\alpha Q\right] = \mathbb{I}[L, M]$,

where the main diagonal elements of W are $w_{ii} = 0, w_{ij} = w_{ji}^*$ for $i \neq j$, and the lower diagonal elements of W are given by

$$\begin{aligned} \left(D_t^\alpha + \overrightarrow{\mho_1} \odot \overrightarrow{\nabla}\right) u_1\left(\overrightarrow{X}, t\right) &= \mathbb{I}\gamma_1 u_2^*\left(\overrightarrow{X}, t\right) u_3^*\left(\overrightarrow{X}, t\right), \\ \left(D_t^\alpha + \overrightarrow{\mho_2} \odot \overrightarrow{\nabla}\right) u_2\left(\overrightarrow{X}, t\right) &= \mathbb{I}\gamma_2 u_1^*\left(\overrightarrow{X}, t\right) u_3^*\left(\overrightarrow{X}, t\right), \\ \left(D_t^\alpha + \overrightarrow{\mho_3} \odot \overrightarrow{\nabla}\right) u_3\left(\overrightarrow{X}, t\right) &= \mathbb{I}\gamma_3 u_1^*\left(\overrightarrow{X}, t\right) u_2^*\left(\overrightarrow{X}, t\right), \end{aligned} \tag{8}$$

in which $\overrightarrow{\mho_i}$ is the velocity vector of the i^{th} wave $u_i\left(\overrightarrow{X}, t\right)$ such that

$$\begin{aligned} \overrightarrow{\mho_1} &= \left(\frac{a_3 b_1 - a_1 b_3}{a_1 - a_3}\right) \quad \mathbf{v} = \mho_{11}\hat{i} + \mho_{12}\hat{j} + \mho_{13}\hat{k}, \\ \overrightarrow{\mho_2} &= \left(\frac{a_2 b_1 - a_1 b_2}{a_1 - a_2}\right) \quad \mathbf{v} = \mho_{21}\hat{i} + \mho_{22}\hat{j} + \mho_{23}\hat{k}, \\ \overrightarrow{\mho_3} &= \left(\frac{a_3 b_2 - a_2 b_3}{a_2 - a_3}\right) \quad \mathbf{v} = \mho_{31}\hat{i} + \mho_{32}\hat{j} + \mho_{33}\hat{k}. \end{aligned} \tag{9}$$

The mathematical model (8) describes the three waves propagating in the space–time with velocity vectors $\overrightarrow{\mho_i}$ at the fractional level $\alpha \in (0,1]$, where the solutions $u_i\left(\overrightarrow{X}, t\right)$ are generally complex-valued functions in which the absolute values of u_i represent the envelope of the i^{th} wave, the propagating waves catch each other at a region called the interaction zone. Anyhow, many interesting physical phenomena have been studied numerically and analytically at the integer level of fractional order $\alpha = 1$ so that the values of γ_i are ±1, which have many physical explanations [21]. The generalization of such model aims to restudy these physical phenomena at different values of fractional order α, which is a not studied mission yet. To achieve our goal and facilitate computations in obtaining exact solutions for the fractional system (8), let

$$\begin{aligned} u_1\left(\overrightarrow{X}, t\right) &= \frac{\mathbb{I}}{\sqrt{\gamma_2\gamma_3}} Q_1\left(\overrightarrow{X}, t\right), \\ u_2\left(\overrightarrow{X}, t\right) &= \frac{\mathbb{I}}{\sqrt{\gamma_1\gamma_3}} Q_2\left(\overrightarrow{X}, t\right), \\ u_3\left(\overrightarrow{X}, t\right) &= \frac{\mathbb{I}}{\sqrt{\gamma_1\gamma_2}} Q_3\left(\overrightarrow{X}, t\right). \end{aligned} \tag{10}$$

Then, the fractional system (8) can be converted into the free parameters fractional system as follows

$$\begin{aligned} \left(D_t^\alpha + \overrightarrow{\mho_1} \odot \overrightarrow{\nabla}\right) Q_1\left(\overrightarrow{X}, t\right) &= -Q_2^*\left(\overrightarrow{X}, t\right) Q_3^*\left(\overrightarrow{X}, t\right), \\ \left(D_t^\alpha + \overrightarrow{\mho_2} \odot \overrightarrow{\nabla}\right) Q_2\left(\overrightarrow{X}, t\right) &= -Q_1^*\left(\overrightarrow{X}, t\right) Q_3^*\left(\overrightarrow{X}, t\right), \\ \left(D_t^\alpha + \overrightarrow{\mho_3} \odot \overrightarrow{\nabla}\right) Q_3\left(\overrightarrow{X}, t\right) &= -Q_1^*\left(\overrightarrow{X}, t\right) Q_2^*\left(\overrightarrow{X}, t\right), \end{aligned} \tag{11}$$

which represents the nonlinear evolution of FTWIEs under the conformable fractional derivative of the fractional level $\alpha \in (0,1]$. For $\alpha = 1$, system (11) is well known and has been studied numerically and analytically. Furthermore, there are many methods used to get solutions for fractional systems of partial differential equations similar to system (11), including Adomian decomposition method, homotopy analysis method, and homotopy Sumudu transform method [41–51].

4. Description of the unified method

In this section, a brief description of the UM is introduced. For more details about this powerful method which is considered the last developed method used to obtain exact solutions for a wide range of nonlinear evolution equations in physics and applied mathematics, we refer to [52–54]. The outlines of the UM are presented as follows.

Consider the fractional system of partial differential equations with conformable time-fractional derivative in the following form

$$P\left(u_i(x_1,x_2,x_3,\cdots,x_n,t),D_t^{\alpha}(u_i),D_t^{2\alpha}(u_i),\cdots,\frac{\partial u_i}{\partial x_j},\frac{\partial^2 u_i}{\partial x_j^2},\frac{\partial^2 u_i}{\partial x_j \partial x_k}\right)=0. \quad (12)$$

According to UM, the assumption $\xi=\left(\sum_{i=1}^{n} m_i x_i\right)-\lambda\frac{t^{\alpha}}{\alpha}$ transform the fractional partial differential system (12) into a system of nonlinear ordinary differential equations of the form

$$U\left(u_i(\xi),u_i'(\xi),u_i''(\xi),\cdots\right)=0, \quad (13)$$

where

$$\begin{array}{lll} D_t^{\alpha}(u_i) &=& -\lambda u_i'(\xi), \qquad D_t^{2\alpha}u_i=\lambda^2 u_i''(\xi),\cdots, \\ \frac{\partial u_i}{\partial x_j} &=& m_j u_i'(\xi), \qquad \frac{\partial^2 u_i}{\partial x_j \partial x_k}=m_j m_k u_i''(\xi). \end{array} \quad (14)$$

Consequently, the rational solution of the fractional partial differential system (12) can be obtained by the following assumption:

$$\begin{array}{l} u_i(\xi)=\left(\sum_{k=0}^{r_1} a_k\phi^k(\xi)\right)\div\left(\sum_{k=0}^{r_2} p_k\phi^k(\xi)\right), \\ \phi'(\xi)=\sum_{k=0}^{r_3}\alpha_k\phi^k(\xi), \end{array} \quad (15)$$

where r_1,r_2,r_3 can be determined by balancing the order of the highest derivative with the highest power of the nonlinear terms appear in the system.

5. Implementing the unified method

The system (11) can be transformed into a system in one variable $\xi\left(\overrightarrow{X},t\right)$ by the assumption:

$$\xi\left(\overrightarrow{X},t\right)=m_1x+m_2y+m_3z-\lambda\frac{t^{\alpha}}{\alpha}, \quad (16)$$

where m_i,λ are constants to be determined. The computations showed that m_i can be chosen arbitrarily, so we can control the studied space, for example if we want to study the problem in the (z,t) space then simply choose $m_1=m_2=0\neq m_3$, while if we want to study the problem in the (x,z,t) space then choose $m_2=0,m_1\neq 0,m_3\neq 0$, and so on. From the constructed function in Eq. (16), it follows $Q_i\left(\overrightarrow{X},t\right)=Q_i(\xi)$. So, by using Theorem 2.1, we have

$$\begin{array}{lll} D_t^{\alpha}Q_i\left(\overrightarrow{X},t\right) &=& -\lambda Q_i'(\xi), \\ \overrightarrow{\nabla}Q_i\left(\overrightarrow{X},t\right) &=& Q_i'(\xi)\,\overrightarrow{\mathbb{M}}, \quad i=1,2,3, \end{array} \quad (17)$$

where $\overrightarrow{\mathbb{M}}=m_1\hat{i}+m_2\hat{j}+m_3\hat{k}$.

By using Eqs. (16) and (17), system (11) becomes as follows

$$\begin{array}{lll} (V_1-\lambda)Q_1'(\xi) &=& -Q_2^*(\xi)Q_3^*(\xi), \\ (V_2-\lambda)Q_2'(\xi) &=& -Q_1^*(\xi)Q_3^*(\xi), \\ (V_3-\lambda)Q_3'(\xi) &=& -Q_1^*(\xi)Q_2^*(\xi), \end{array} \quad (18)$$

where $V_i=\overrightarrow{\mho_i}\odot\overrightarrow{\mathbb{M}}$. If we balance the order of the highest derivative with the highest power of the nonlinear terms appear in Eq. (18), we find $\{r_1,r_2,r_3\}=\{2,1,2\}$, and thus the second part of Eq. (15) becomes as follows

$$\phi'(\xi) = \alpha_0+\alpha_1\phi(\xi)+\alpha_2\phi^2(\xi). \quad (19)$$

Hence, we have

$$\begin{array}{l} \phi(\xi)=\frac{-\alpha_1+\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}\tan\left(\frac{1}{2}(\xi+c)\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}\right)}{2\alpha_2}, \\ Q_1(\xi)=\frac{A_0+A_1\phi(\xi)+A_2\phi^2(\xi)}{p_0+p_1\phi(\xi)}, \\ Q_2(\xi)=\frac{B_0+B_1\phi(\xi)+B_2\phi^2(\xi)}{q_0+q_1\phi(\xi)}, \\ Q_3(\xi)=\frac{C_0+C_1\phi(\xi)+C_2\phi^2(\xi)}{r_0+r_1\phi(\xi)}, \end{array} \quad (20)$$

where the undetermined constants $\{A_j,B_j,C_j\},j=0,1,2$, are complex numbers and $\{p_j,q_j,r_j\},j=0,1$, are chosen to be real numbers. The computations showed that $\{\alpha_j,c\}$ are arbitrary non zero real constants and must be chosen so that $-\alpha_1^2+4\alpha_0\alpha_2>0$. When substituting Eq. (20) into Eq. (18), we get the following set of equations:

$$\begin{array}{l} (V_1-\lambda)(q_0+q_1\phi)(r_0+r_1\phi)(A_1p_0-A_0p_1+A_2\phi(2p_0+p_1\phi))\phi'+ \\ (p_0+p_1\phi)^2\left(B_0^*+B_1^*\phi+B_2^*\phi^2\right)\left(C_0^*+C_1^*\phi+C_2^*\phi^2\right)=0, \end{array} \quad (21)$$

$$\begin{array}{l} (V_2-\lambda)(p_0+p_1\phi)(r_0+r_1\phi)(B_1q_0-B_0q_1+B_2\phi(2q_0+q_1\phi))\phi'+ \\ (q_0+q_1\phi)^2\left(A_0^*+A_1^*\phi+A_2^*\phi^2\right)\left(C_0^*+C_1^*\phi+C_2^*\phi^2\right)=0, \end{array} \quad (22)$$

$$\begin{array}{l} (V_3-\lambda)(p_0+p_1\phi)(q_0+q_1\phi)(C_1r_0-C_0r_1+C_2\phi(2r_0+r_1\phi))\phi'+ \\ (r_0+r_1\phi)^2\left(A_0^*+A_1^*\phi+A_2^*\phi^2\right)\left(B_0^*+B_1^*\phi+B_2^*\phi^2\right)=0. \end{array} \quad (23)$$

If we collect the coefficients of $\{\phi^k:k=0,1,\cdots 6\}$ and make these coefficients equal zero, we get a system of 21 equations with 19 unknowns free parameters, namely $\{\lambda,A_j,B_j,C_j,\alpha_j:j=0,1,2\}$ and $\{p_k,q_k,r_k:k=0,1\}$. Subsequently, the system resulted from Eqs. (21)–(23) has many different sets of solutions that could be found by many mathematical softwares such as Mathematica 12. Moreover, if we use any of the obtained solutions of the system resulted from Eqs. (21)–(23) then substitute this solution in (20) to get a solution for the proposed problem (18). To do so, we follow either Algorithm 5.1 or 5.2 stated as follows:

Algorithm 5.1: In order to obtain solutions of the system resulted from Eqs. (21)–(23), do the following steps:

Step (1): Construct the vectors $\overrightarrow{\mho_i}$ in Eq. (9).

Step (2): Determine the study space, then choose arbitrary real numbers m_i to construct the vector $\overrightarrow{\mathbb{M}}$ in Eq. (17) and compute $V_i=\overrightarrow{\mho_i}\odot\overrightarrow{\mathbb{M}}$.

Step (3): Choose 3 arbitrary non zero real numbers $\alpha_0,\alpha_1,\alpha_2$ so that $-\alpha_1^2+4\alpha_0\alpha_2>0$.

Step (4): Choose a real number $\lambda>\mathrm{Max}\{V_1,V_2,V_3\}$.

Step (5): Choose $k_1,l_1\in[-1,1]$, and compute $\{K,L,M\}$ from one of the following sets:

$$\begin{aligned}
&\left\{K=-\mathbb{I}, L=l_1+\mathbb{I}\sqrt{1-l_1^2}, M=\mathbb{I}l_1+\sqrt{1-l_1^2}\right\},\\
&\left\{K=-\mathbb{I}, L=l_1-\mathbb{I}\sqrt{1-l_1^2}, M=\mathbb{I}l_1-\sqrt{1-l_1^2}\right\},\\
&\left\{K=\mathbb{I}, L=l_1+\mathbb{I}\sqrt{1-l_1^2}, M=-\mathbb{I}l_1-\sqrt{1-l_1^2}\right\},\\
&\left\{K=\mathbb{I}, L=l_1-\mathbb{I}\sqrt{1-l_1^2}, M=-\mathbb{I}l_1+\sqrt{1-l_1^2}\right\},\\
&\left\{K=k_1+\mathbb{I}\sqrt{1-k_1^2}, L=l_1+\mathbb{I}\sqrt{1-l_1^2},\right.\\
&\left.M=k_1l_1-\sqrt{1-k_1^2}\sqrt{1-l_1^2}+\mathbb{I}\left(-\sqrt{1-k_1^2}l_1-k_1\sqrt{1-l_1^2}\right)\right\},\\
&\left\{K=k_1+\mathbb{I}\sqrt{1-k_1^2}, L=l_1-\mathbb{I}\sqrt{1-l_1^2},\right.\\
&\left.M=k_1l_1+\sqrt{1-k_1^2}\sqrt{1-l_1^2}+\mathbb{I}\left(-\sqrt{1-k_1^2}l_1+k_1\sqrt{1-l_1^2}\right)\right\},\\
&\left\{K=k_1-\mathbb{I}\sqrt{1-k_1^2}, L=l_1+\mathbb{I}\sqrt{1-l_1^2},\right.\\
&\left.M=k_1l_1+\sqrt{1-k_1^2}\sqrt{1-l_1^2}+\mathbb{I}\left(\sqrt{1-k_1^2}l_1-k_1\sqrt{1-l_1^2}\right)\right\},\\
&\left\{K=k_1-\mathbb{I}\sqrt{1-k_1^2}, L=l_1-\mathbb{I}\sqrt{1-l_1^2},\right.\\
&\left.M=k_1l_1-\sqrt{1-k_1^2}\sqrt{1-l_1^2}+\mathbb{I}\left(\sqrt{1-k_1^2}l_1+k_1\sqrt{1-l_1^2}\right)\right\}.
\end{aligned}\tag{24}$$

Step (6): The solutions $Q_i(\xi)$ in Eq. (20) will be simplified to the following form:

$$\begin{aligned}
Q_1(\xi) &= K\sqrt{\lambda-V_2}\sqrt{\lambda-V_3}\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}f_1(\xi),\\
Q_2(\xi) &= L\sqrt{\lambda-V_1}\sqrt{\lambda-V_3}\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}f_2(\xi),\\
Q_3(\xi) &= M\sqrt{\lambda-V_1}\sqrt{\lambda-V_2}\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}f_3(\xi),
\end{aligned}\tag{25}$$

in which $\{f_1(\xi), f_2(\xi), f_3(\xi)\}$ can be considered as one of the following sets:

$$\begin{aligned}
&\left\{\tan\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right), \sec\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right),\right.\\
&\left.\sec\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right)\right\},\\
&\left\{\sec\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right), \tan\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right),\right.\\
&\left.\sec\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right)\right\},\\
&\left\{\sec\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right), \sec\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right),\right.\\
&\left.\tan\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right)\right\},
\end{aligned}\tag{26}$$

where c is arbitrary real constant.

Step (7): The obtained functions in Eq. (25) are solutions for system (11) after using the assumption given in Eq. (16).

Another form of solutions for the proposed problem (18) could be found by doing the following algorithm too:

Algorithm 5.2: In order to obtain another set of solutions of the proposed system, do the following steps:

Step (1): Do the first 4 steps as we did in Algorithm 5.1.

Step (2): Choose $k_1, l_1 \in [-1, 1]$, and compute $\{K, L, M\}$ from one of the following sets:

$$\begin{aligned}
&\left\{K=\mathbb{I}, L=l_1+\mathbb{I}\sqrt{1-l_1^2}, M=\mathbb{I}l_1+\sqrt{1-l_1^2}\right\},\\
&\left\{K=\mathbb{I}, L=l_1-\mathbb{I}\sqrt{1-l_1^2}, M=\mathbb{I}l_1-\sqrt{1-l_1^2}\right\},\\
&\left\{K=-\mathbb{I}, L=l_1+\mathbb{I}\sqrt{1-l_1^2}, M=-\mathbb{I}l_1-\sqrt{1-l_1^2}\right\},\\
&\left\{K=-\mathbb{I}, L=l_1-\mathbb{I}\sqrt{1-l_1^2}, M=-\mathbb{I}l_1+\sqrt{1-l_1^2}\right\},\\
&\left\{K=k_1+\mathbb{I}\sqrt{1-k_1^2}, L=l_1+\mathbb{I}\sqrt{1-l_1^2},\right.\\
&\left.M=-k_1l_1+\sqrt{1-k_1^2}\sqrt{1-l_1^2}+\mathbb{I}\left(\sqrt{1-k_1^2}l_1+k_1\sqrt{1-l_1^2}\right)\right\},\\
&\left\{K=k_1+\mathbb{I}\sqrt{1-k_1^2}, L=l_1-\mathbb{I}\sqrt{1-l_1^2},\right.\\
&\left.M=-k_1l_1-\sqrt{1-k_1^2}\sqrt{1-l_1^2}+\mathbb{I}\left(\sqrt{1-k_1^2}l_1-k_1\sqrt{1-l_1^2}\right)\right\},\\
&\left\{K=k_1-\mathbb{I}\sqrt{1-k_1^2}, L=l_1+\mathbb{I}\sqrt{1-l_1^2},\right.\\
&\left.M=-k_1l_1-\sqrt{1-k_1^2}\sqrt{1-l_1^2}+\mathbb{I}\left(-\sqrt{1-k_1^2}l_1+k_1\sqrt{1-l_1^2}\right)\right\},\\
&\left\{K=k_1-\mathbb{I}\sqrt{1-k_1^2}, L=l_1-\mathbb{I}\sqrt{1-l_1^2},\right.\\
&\left.M=-k_1l_1+\sqrt{1-k_1^2}\sqrt{1-l_1^2}+\mathbb{I}\left(-\sqrt{1-k_1^2}l_1-k_1\sqrt{1-l_1^2}\right)\right\}.
\end{aligned}\tag{27}$$

Step (3): The solutions $Q_i(\xi)$ given in Eq. (20) will be simplified to the following form:

$$\begin{aligned}
Q_1(\xi) &= K\sqrt{\lambda-V_2}\sqrt{\lambda-V_3}\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}f_1(\xi),\\
Q_2(\xi) &= L\sqrt{\lambda-V_1}\sqrt{\lambda-V_3}\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}f_2(\xi),\\
Q_3(\xi) &= M\sqrt{\lambda-V_1}\sqrt{\lambda-V_2}\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}f_3(\xi),
\end{aligned}\tag{28}$$

in which $\{f_1(\xi), f_2(\xi), f_3(\xi)\}$ can be considered as one of the following sets in Eqs. (29) and (30):

$$\begin{aligned}
&\left\{\cot\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right), \csc\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right),\right.\\
&\left.\csc\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right)\right\},\\
&\left\{\csc\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right), \cot\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right),\right.\\
&\left.\csc\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right)\right\},\\
&\left\{\csc\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right), \csc\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right),\right.\\
&\left.\cot\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right)\right\},
\end{aligned}\tag{29}$$

or

$$\begin{aligned}
&\left\{coth\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right), \mathrm{csch}\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right),\right.\\
&\left.\mathrm{csch}\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right)\right\},\\
&\left\{\mathrm{csch}\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right), coth\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right),\right.\\
&\left.\mathrm{csch}\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right)\right\},\\
&\left\{\mathrm{csch}\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right), \mathrm{csch}\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right),\right.\\
&\left.coth\left(\sqrt{-\alpha_1^2+4\alpha_0\alpha_2}(\xi+c)\right)\right\}.
\end{aligned}\tag{30}$$

Step (4): The obtained functions in Eq. (28) are solutions of Eq. (11) after using the assumption in Eq. (16)

6. Applications and examples

In this section, the applicability and accuracy of the proposed method are demonstrated by including some examples for the modified FTWIEs. More specifically, Algorithms 5.1 and 5.2 are applied to get some rational solutions for the fractional system (11). All computations are performed using wolfram Mathematica 12.

Example 6.1: Let us consider the special solutions of the system resulted from Eqs. (21)–(23) as follows:

$$A_0 = -\alpha_0 p_1\sqrt{\lambda - V_2}\sqrt{\lambda - V_3}, \quad A_1 = -\alpha_1 p_1\sqrt{\lambda - V_2}\sqrt{\lambda - V_3},$$
$$A_2 = -\alpha_2 p_1\sqrt{\lambda - V_2}\sqrt{\lambda - V_3},$$

$$B_0 = -i\alpha_0 q_1\sqrt{\lambda - V_1}\sqrt{\lambda - V_3}, \quad B_1 = -i\alpha_1 q_1\sqrt{\lambda - V_1}\sqrt{\lambda - V_3},$$
$$B_2 = -i\alpha_2 q_1\sqrt{\lambda - V_1}\sqrt{\lambda - V_3},$$
$$C_0 = -\frac{i\left(\alpha_1^2 - 2\alpha_0\alpha_2\right) r_1\sqrt{\lambda - V_1}\sqrt{\lambda - V_2}}{2\alpha_2}, \quad C_1 = -i\alpha_1 r_1\sqrt{\lambda - V_1}\sqrt{\lambda - V_2},$$
$$C_2 = -i\alpha_2 r_1\sqrt{\lambda - V_1}\sqrt{\lambda - V_2},$$
$$p_0 = \frac{\alpha_1 p_1}{2\alpha_2}, \quad q_0 = \frac{\alpha_1 q_1}{2\alpha_2}, \quad \text{and} \quad r_0 = \frac{\alpha_1 r_1}{2\alpha_2}. \tag{31}$$

Following the UM, the solution of FTWIEs (11) can be obtained by substituting the values of Eq. (31) into Eq. (20) and using Eq. (16) such that

$$\begin{aligned} Q_1\left(\overrightarrow{X},t\right) &= -\sqrt{\lambda - V_2}\sqrt{\lambda - V_3}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\csc(\theta), \\ Q_2\left(\overrightarrow{X},t\right) &= -i\sqrt{\lambda - V_1}\sqrt{\lambda - V_3}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\csc(\theta), \\ Q_3\left(\overrightarrow{X},t\right) &= i\sqrt{\lambda - V_1}\sqrt{\lambda - V_2}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\cot(\theta), \end{aligned} \tag{32}$$

where $\theta = \sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\left(m_1 x + m_2 y + m_3 z - \lambda\frac{t^\alpha}{\alpha} + c\right)$ and c is an arbitrary real number.

However, the solution given in Eq. (32) can be obtained also by choosing $\{k_1, l_1\} = \{-1, 0\}$ at the 6th set in Eq. (27) and the 3rd set in Eq. (29). On the other hand, the following parameters used to produce Fig. 1 are chosen randomly so that they satisfied all restrictions mentioned in Algorithm 5.2:

$$\begin{aligned} \{a_1, a_2, a_3\} &= \{2.27814, 1.6048, 1.3385\}, \\ \{b_1, b_2, b_3\} &= \{2.90658, 2.19972, 1.37289\}, \\ \overrightarrow{v} &= \{1.22911, 1.97913, 1.59877\}, \\ \overrightarrow{\mho_1} &= \{0.997807, 1.60668, 1.2979\}, \\ \overrightarrow{\mho_2} &= \{-0.633025\}, -1.0193, -0.823411\}, \\ \overrightarrow{\mho_3} &= \{3.42055, 5.50779, 4.4493\}, \\ \overrightarrow{\mathbb{M}} &= \{1, 0, 0\}, \\ \{V_1, V_2, V_3\} &= \{0.997807, -0.633025, 3.42055\}, \\ \{\alpha_0, \alpha_1, \alpha_2\} &= \{3.08964, 4.94153, 2.85647\}, \\ \{\lambda, k_1, l_1\} &= \{3.92055, 0, 1\}. \end{aligned} \tag{33}$$

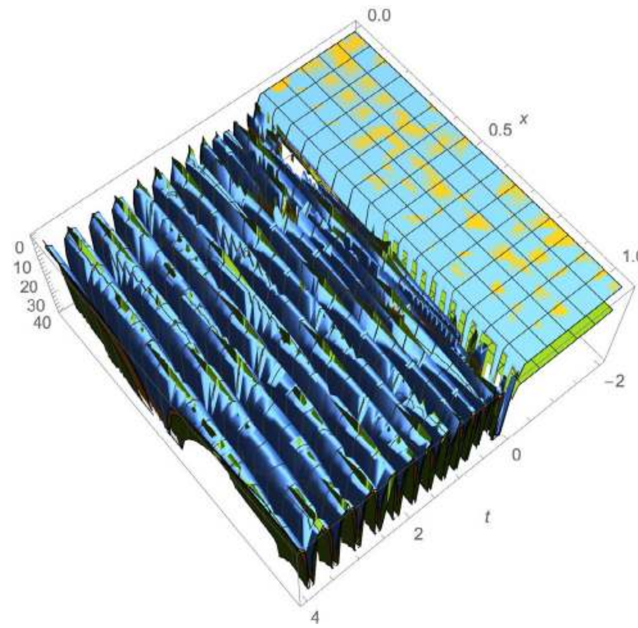

Fig. 1 3D plot of complex solutions, $Q_i(x,t), i = 1,2,3,$ of Example 6.1 at $\alpha = 0.5$.

Consequently, the solution of FTWIEs (11) becomes:

$$\begin{aligned} Q_1(x,t) &= (4.97779\, i)\csc\left(3.29895\left(-7.84109\sqrt{t} + x\right)\right), \\ Q_2(x,t) &= 3.988\csc\left(3.29895\left(-7.84109\sqrt{t} + x\right)\right), \\ Q_3(x,t) &= (12.035\, i)\cot\left(3.29895\left(-7.84109\sqrt{t} + x\right)\right). \end{aligned} \tag{34}$$

Example 6.2: Let us consider $\left\{k_1 = \frac{1}{2}, l_1 = \frac{-1}{2}\right\}$ at the 5^{th} set of Eq. (24) so that $\left\{K = \frac{1}{2}\left(1 + \sqrt{3}i\right),\ L = \frac{1}{2}\left(-1 + \sqrt{3}i\right),\ M = -1\right\}$. Thus, by substituting the resulting values of $\{K, L, M\}$ in Eq. (25) and utilizing Eqs. (16) and (26), the solutions of FTWIEs (11) can be obtained as follows:

$$\begin{aligned} Q_1\left(\overrightarrow{X},t\right) &= \left(\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}i\right)\sqrt{\lambda - V_2}\sqrt{\lambda - V_3}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\tan(\theta), \\ Q_2\left(\overrightarrow{X},t\right) &= \left(\tfrac{-1}{2} + \tfrac{\sqrt{3}}{2}i\right)\sqrt{\lambda - V_1}\sqrt{\lambda - V_3}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\sec(\theta), \\ Q_3\left(\overrightarrow{X},t\right) &= -\sqrt{\lambda - V_1}\sqrt{\lambda - V_2}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\sec(\theta), \end{aligned} \tag{35}$$

or

$$\begin{aligned} Q_1\left(\overrightarrow{X},t\right) &= \left(\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}i\right)\sqrt{\lambda - V_2}\sqrt{\lambda - V_3}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\sec(\theta), \\ Q_2\left(\overrightarrow{X},t\right) &= \left(\tfrac{-1}{2} + \tfrac{\sqrt{3}}{2}i\right)\sqrt{\lambda - V_1}\sqrt{\lambda - V_3}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\tan(\theta), \\ Q_3\left(\overrightarrow{X},t\right) &= -\sqrt{\lambda - V_1}\sqrt{\lambda - V_2}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\sec(\theta), \end{aligned} \tag{36}$$

or

$$\begin{aligned} Q_1\left(\overrightarrow{X},t\right) &= \left(\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}i\right)\sqrt{\lambda - V_2}\sqrt{\lambda - V_3}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\sec(\theta), \\ Q_2\left(\overrightarrow{X},t\right) &= \left(\tfrac{-1}{2} + \tfrac{\sqrt{3}}{2}i\right)\sqrt{\lambda - V_1}\sqrt{\lambda - V_3}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\sec(\theta), \\ Q_3\left(\overrightarrow{X},t\right) &= -\sqrt{\lambda - V_1}\sqrt{\lambda - V_2}\sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\tan(\theta), \end{aligned} \tag{37}$$

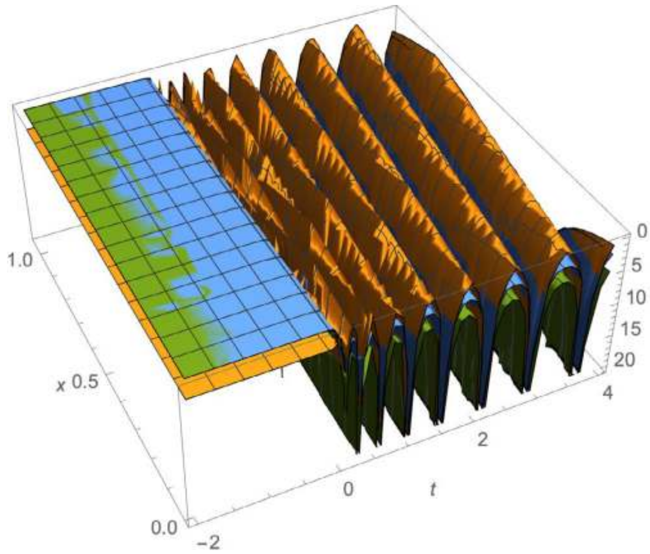

Fig. 2 3D plot of complex solutions, $Q_i(x,t), i = 1,2,3,$ of Example 6.2 at $\alpha = 0.5$.

where $\theta = \sqrt{-\alpha_1^2 + 4\alpha_0\alpha_2}\left(m_1x + m_2y + m_3z - \lambda\frac{t^\alpha}{\alpha} + c\right)$ and c is an arbitrary real number.

Furthermore, the following parameters used to produce Fig. 2 are chosen randomly so that they satisfied all restrictions mentioned in Algorithm 5.2:

$$\begin{aligned}
&\{a_1,a_2,a_3\} = \{2.554, 1.71491, 1.48021\}\\
&\{b_1,b_2,b_3\} = \{2.79625, 1.75043, 1.31935\}\\
&\overrightarrow{v} = \{1.18605, 1.80282, 1.27734\}\\
&\overrightarrow{\mho_1} = \{0.84983, 1.29175, 0.91524\},\\
&\overrightarrow{\mho_2} = \{0.459008, 0.697699, 0.494337\},\\
&\overrightarrow{\mho_3} = \{1.6596, 2.52261, 1.78733\},\\
&\overrightarrow{\mathbb{M}} = \{1,0,0\},\\
&\{V_1,V_2,V_3\} = \{0.84983, 0.459008, 1.6596\},\\
&\{\alpha_0,\alpha_1,\alpha_2\} = \{2.61536, 4.66082, 3.06314\},\\
&\{\lambda,K,L,M\} = \left\{2.1596, \tfrac{1}{2}\left(1+\mathbb{I}\sqrt{3}\right), \tfrac{1}{2}\left(-1+\mathbb{I}\sqrt{3}\right), -1\right\}.
\end{aligned} \tag{38}$$

Consequently, the solution of FTWIEs (11) becomes:

$$\begin{aligned}
Q_1(x,t) &= (1.48125 + 2.5656\,\mathbb{I})\tan\left(3.21273\left(-4.31919\sqrt{t} + x\right)\right),\\
Q_2(x,t) &= (-1.29995 + 2.25158\,\mathbb{I})\sec\left(3.21273\left(-4.31919\sqrt{t} + x\right)\right),\\
Q_3(x,t) &= -4.7948\sec\left(3.21273\left(-4.31919\sqrt{t} + x\right)\right).
\end{aligned} \tag{39}$$

Figs. 1 and 2 are the 3D graphs of Examples 6.1 and 6.2 respectively, which represent the absolute values of the complex solutions, $Q_i(x,t), i = 1,2,3$, where the order of the fractional derivative is chosen as $\alpha = 0.5$. From these figures, we notice that each slice represents the surface of the three-wave packets, and some of these waves separate in more than one hump, which is a nature of the soliton waves during the interaction. Also, we observe that the interaction between the three waves takes place after $t > 0$, while the three waves move separately before the interaction started since they move with different velocities.

7. Conclusion

In this paper, the unified method has been successfully applied for obtaining solutions for a class of nonlinear fractional evolution equations. A modification of Lax-pair operators based on Zakharov-Manakov concept was used to construct generalization of three wave interaction equations in the sense of conformable fractional derivative. Systematic algorithms have been given to get an infinite set of solutions for the proposed system. By randomly choosing specific values of free parameters, the exact rational solutions have been shown different patterns. Our obtained solutions can be easily switched to be solutions for the same problem but in one or two dimensions instead of three dimensions. Also, We are sure that there are more interested solutions which could be found by the same method, so we keep this as a future work as well as the study of the behavior of these solutions. We think that the obtained solutions are useful and lead to many applications in the study of wave propagating phenomena.

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Declaration of Competing Interest

The authors declare no conflict of interest.

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