

Infinite Networks of Identical Capacitors

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Abstract

The capacitance between the origin and any other lattice site in an infinite square lattice of identical capacitors each of capacitance C is calculated. The method is generalized to infinite Simple Cubic (SC) lattice of identical capacitors each of capacitance C . We make use of the superposition principle and the symmetry of the infinite grid.

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1. Introduction

A classic problem in electric circuit theory studied by many authors over many years is computation of the resistance between two nodes in a resistor network. Besides being a central problem in electric circuit theory, the computation of resistances is also relevant to a wide range of problems ranging from random walk^{1,2}, theory of harmonic functions³ to first-passage processes⁴ to Lattice Green's Functions⁵ (LGF). The connection with these problems originates from the fact that electrical potentials on a grid are governed by the same difference equations as those occurring in the other problems. For this reason, the resistance problem is often studied from the point of view of solving the difference equations, which is most conveniently carried out for infinite networks. Kirchhoff⁶ formulated the study of electric networks more than 150 years ago. The electric-circuit theory is discussed in detail in a classic text by Van der Pol and Bremmer⁷ where they derived the resistance between nearby points on the square lattice.

In the previous 60 years many efforts were focused on analyzing infinite networks of identical resistors. In these efforts scientists used different methods. For example, Francis J. Bartis⁸ introduced how complex systems can be treated at the elementary level and showed how to calculate the effective resistance between adjacent nodes of different lattices of one-ohm resistors. His note is interesting and educational. Venezian⁹, Atkinson and Van Steenwijk¹⁰ used the superposition of current distribution to calculate the effective resistance between adjacent sites on an infinite networks (i.e. Square, SC, Hexagonal,...). The mathematical problem involves the solution of an infinite set of linear, inhomogeneous difference equations which are solved by the method of separation of variables. Numerical results for the resistances between arbitrary sites are presented.

Monwhea Jeng¹¹ introduced a mapping between random walk problems and resistor network problems, where his method was used to calculate the effective resistance between any two sites in an infinite two-dimensional square lattice of unit resistors and the superposition principle was used to find the effective resistances on toroidal- and cylindrical-square- lattices. In the last decade many papers¹²⁻¹⁸ have been published using an alternative method based on the LGF. The resistance between two arbitrary lattice sites for different infinite networks of identical resistors were studied for both the perfect and the perturbed networks. Finally, Wu¹⁹ obtained the resistance between two arbitrary nodes in a resistor network in terms of the eigenvalues and eigenfunctions of the Laplacian matrix associated with the network. Explicit formulae for two

point resistances are deduced in his paper for regular lattices in one, two, and three- dimensions under various boundary conditions.

Little attention has been paid to infinite networks consisting of identical capacitances C . Van Enk²⁰ studied the behavior of the impedance of a standard ladder network of capacitors and inductors where he analyzed it as a function of the size of the network. Recently, Asad et al^{21 - 23} and Hijjawi et al²⁴ investigated many infinite lattices of identical capacitors using the LGF method and Charge distribution method. In these papers numerical results for the equivalent capacitance between the origin and any other lattice site was presented-using the above two methods- for the perfect infinite square network. Numerical results was also presented for the so-called perturbed infinite square network which results by removing one or two bonds from the perfect network.

In this paper we investigate analytically and numerically the capacitance between arbitrary lattice sites in an infinite square and SC grids using the charge distribution method which is based upon the superposition principle. The asymptotic behavior is also studied for large separation between the two sites. The basic approach used here is similar to that followed by Atkinson and Steenwijk¹⁰.

Finally, it is important to mention that problems involving large or infinite electrical resistive networks are interesting and educational^{25, 26}. Applications include geophysical exploration with electrical currents, petroleum flow in oil wells, and random walks²⁷.

2. Infinite Square Lattice

In this section, we consider an infinite square network consisting of identical capacitances C . Let us define the voltage at the node (l, m) to be given by $V_{l, m}$, and suppose that a charge of $Q_{l, m}$ enters that node from an external source.

Now using Ohm's and Kirchhoff's laws, we can write:

$$Q_{l, m} = (V_{l, m} - V_{l+1, m})C + (V_{l, m} - V_{l-1, m})C + (V_{l, m} - V_{l, m+1})C + (V_{l, m} - V_{l, m-1})C. \quad (2.1)$$

$$= 4V_{l, m}C - V_{l+1, m}C - V_{l-1, m}C - V_{l, m+1}C - V_{l, m-1}C. \quad (2.2)$$

We shall look for an integral representation for $V_{l, m}$, and take it to be in the form:

$$V_{l, m} = \int_0^{2\pi} d\beta F(\beta) V_{l, m}(\beta). \quad (2.3)$$

with

$$V_{l,m}(\beta) = \exp(i|l|\alpha + im\beta). \quad (2.4)$$

Here α is a function of β .

The above representation is a modified Fourier transform.

For $l > 0$, we get:

$$4V_{l,m}(\beta) - V_{l+1,m}(\beta) - V_{l,m+1}(\beta) - V_{l,m-1}(\beta) = 2 \exp(i|l|\alpha + im\beta)[2 - \text{Cos}\alpha - \text{Cos}\beta]. \quad (2.5)$$

From the above equation, we require (i.e. in order that the contributions of the potential vanishes) α to be related to β as:

$$\text{Cos}\alpha + \text{Cos}\beta = 2. \quad (2.6)$$

In a similar way, we find zero for this contribution if $l < 0$. Thus, for any integrable $F(\beta)$, then $Q_{l,m}$ given in Eq. (2.1) goes to zero, unless $l = 0$.

For $0 < \beta < 2\pi$, then Eq.(2.6) has only an imaginary solution given by:

$$\alpha = i \text{Log}[2 - \text{Cos}\beta + \sqrt{3 - 4\text{Cos}\beta + \text{Cos}^2\beta}]. \quad (2.7)$$

For the case $l = 0$, we may write:

$$Q_{0,m} = C \int_0^{2\pi} d\beta F(\beta) \exp(im\beta)[4 - 2 \exp(i\alpha) - 2\text{Cos}\beta];$$

$$= 2C \int_0^{2\pi} d\beta F(\beta) \exp(im\beta)[\text{Cos}\alpha - \exp(i\alpha)];$$

$$= -2iC \int_0^{2\pi} d\beta F(\beta) \text{Sin}\alpha \exp(im\beta). \quad (2.8)$$

The above charges may be construed as the coefficients of the Fourier series

$$-2iF(\beta)\text{Sin}\alpha = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} Q_{0,m} \exp(-im\beta). \quad (2.9)$$

Now, take $Q_{0,m} = \delta_{0m}$; i.e. $Q_{0,0} = 1$, and $Q_{0,m} = 0$. This situation correspond to the case where a charge Q enters at the node $(0,0)$ and leaves at infinity. Note that no charges leave the lattice at any other finite node. So, with this choice

$$F(\beta) = \frac{i}{4\pi \text{Sin}\alpha}. \quad (2.10)$$

Thus, we can write:

$$V_{l,m} = \frac{i}{4\pi} \int_0^{2\pi} \frac{d\beta}{\text{Sin}\alpha} \exp(i|l|\alpha + im\beta). \quad (2.11)$$

One may ask. What is the capacitance between the origin and the site (l,m) ? First of all, it is clear that the potential difference between these two sites is $(V_{0,0} - V_{l,m})$. Now, imagine that a charge of one micro Coulomb enters the network at the node (l,m) instead of $(0,0)$, allowing it to leave at infinity. The new potential at (l,m) will now be what we called $V_{0,0}$ and the new potential at $(0,0)$ will be, by symmetry, what we called $V_{l,m}$. Thus, the new potential difference between the origin and (l,m) is just minus the previous one.

If we choose to imagine that one micro Coulomb leaves the node (l,m) , so that all the potential difference will be reversed in sign. Therefore, the new potential difference between the origin and the node (l,m) is again given by $(V_{0,0} - V_{l,m})$.

Exploiting the linearity of Ohm's law and superpose all the charges and potentials appertaining to the configuration in which one micro Coulomb enters at $(0,0)$ and leaves at (l,m) , one can write:

$$2[V_{0,0} - V_{l,m}]C_{l,m} = 1. \quad (2.12)$$

Or,

$$C_{l,m} = \frac{1}{2[V_{0,0} - V_{l,m}]}. \quad (2.13)$$

Thus, $C_{l,m}$ can be written as:

$$C_{l,m} = \frac{1}{\frac{i}{2\pi} \int_0^{2\pi} \frac{d\beta}{\text{Sin}\alpha} [1 - \exp(i|l|\alpha + im\beta)]}. \quad (2.14)$$

It is obvious that $C_{l,m} = C_{m,l}$ due to the symmetry of the lattice. Finally, we may transform Eq. (2.14) into the manifestly real form:

$$C_{l,m} = \frac{1}{\frac{1}{\pi} \int_0^{\pi} \frac{d\beta}{\text{Sinh}|\alpha|} [1 - \exp(-|l|\alpha) \text{Cos}m\beta]}. \quad (2.15)$$

$$\text{As } l \rightarrow \infty, C_{\infty, m} \rightarrow \frac{1}{\frac{1}{\pi} \int_0^{\pi} \frac{d\beta}{\text{Sinh}|\alpha|}} \rightarrow 0.$$

Using Eq. (2.15), one can calculate the capacitance $C_{l,m}$ by means of Mathematica. Table 1 below shows some calculated values. Similar results were obtained by us²² using similar approach, and by using LGF approach⁷.

3. Infinite SC Network

The above method can be generalized to an infinite three - dimensional SC network of identical capacitors. Here we consider three indices and a charge entering the site (l,m,n) is related to the potentials by:

$$Q_{l,m,n} = (6V_{l,m,n} - V_{l+1,m,n} - V_{l-1,m,n} - V_{l,m+1,n} - V_{l,m-1,n} - V_{l,m,n+1} - V_{l,m,n-1})C. \quad (3.1)$$

Choose $V_{l,m,n}$ to be given as:

$$V_{l,m,n} = \int_0^{2\pi} d\beta \int_0^{2\pi} d\gamma F(\beta, \gamma) v_{l,m,n}(\beta, \gamma). \quad (3.2)$$

Where $v_{l,m,n}(\beta, \gamma) = \exp(i|l|\alpha + im\beta + in\gamma)$, and with $\text{Cos}\alpha + \text{Cos}\beta + \text{Cos}\gamma = 3$.

i.e.; $\alpha = \text{Cos}^{-1}(3 - \text{Cos}\beta - \text{Cos}\gamma)$.

For $l \neq 0$, one can easily show that:

$$Q_{l,m,n} = 2 \int_0^{2\pi} d\beta \int_0^{2\pi} d\gamma F(\beta, \gamma) \exp(i|l|\alpha + im\beta + in\gamma) x [3 - \text{Cos}\alpha - \text{Cos}\beta - \text{Cos}\gamma]. \quad (3.3)$$

whereas:

$$Q_{0,m,n} = -2i \int_0^{2\pi} d\beta \int_0^{2\pi} d\gamma F(\beta, \gamma) \text{Sin}\alpha \text{Cos}m\beta \text{Cos}n\gamma. \quad (3.4)$$

The inverse of this double Fourier series gives:

$$-2iF(\beta, \gamma) \text{Sin}\alpha = \frac{4}{\pi^2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Q_{0,m,n} \exp(-im\beta) \exp(-in\gamma). \quad (3.5)$$

Let us choose $Q_{0,0,0} = 1$, and $Q_{0,m,n} = 0$, unless both m and n vanish. This implies that:

$$F(\beta, \gamma) = \frac{i}{8\pi^2 \text{Sin} \alpha}. \quad (3.6)$$

Substitute Eq. (3.6) into Eq. (3.2), it yields the potential $V_{l,m,n}$. That is,

$$V_{l,m,n} = \int_0^{2\pi} d\beta \int_0^{2\pi} d\gamma \frac{i}{8\pi^2 \text{Sin} \alpha} v_{l,m,n}(\beta, \gamma). \quad (3.7)$$

As in section 2, we can compute the capacitance $C_{l,m,n}$ by assuming that a charge of one micro Coulomb enters the origin and leaves the lattice site (l,m,n) . Thus;

$$C_{l,m,n} = \frac{1}{\frac{i}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \frac{d\beta d\gamma}{\text{Sin} \alpha} [1 - \exp(i|l|\alpha) + im\beta + in\gamma]}. \quad (3.8)$$

Again, this expression is symmetric under any permutation of the indices. A manifestly real form of Eq. (3.8) is:

$$C_{l,m,n} = \frac{1}{\frac{1}{\pi^2} \int_0^\pi \int_0^\pi \frac{d\beta d\gamma}{\text{Sinh}|\alpha|} [1 - \exp(-|l|\alpha) \text{Cos} m\beta \text{Cos} n\gamma]}. \quad (3.9)$$

$$\text{As } l \rightarrow \infty, C_{\infty,m,n} \rightarrow \frac{1}{\frac{1}{\pi^2} \int_0^\pi \int_0^\pi \frac{d\beta d\gamma}{\text{Sinh}|\alpha|}} \rightarrow 0.$$

Using Eq. (3.9), we can calculate $C_{l,m,n}$ by means of Mathematica. Table 2 below shows some calculated values.

This method can be straight forwardly generalized to four and more dimensions. In a $(d+1)$ dimensional hypercubic lattice, the capacitance between the origin and the lattice site (m_1, m_2, \dots, m_d) is

$$C_{m_1, \dots, m_d} = \frac{1}{\frac{i}{(2\pi)^d} \int_0^{2\pi} \dots \int_0^{2\pi} \frac{d\beta_1 \dots d\beta_d}{\text{Sin} \alpha} [1 - \exp(i|m_1|\alpha) + im_2\beta_2 + \dots + im_d\beta_d]}. \quad (3.10)$$

Where $\text{Cos} \alpha + \text{Cos} \beta_1 + \dots + \text{Cos} \beta_d = d$.

As a further generalization, consider the SC network with different capacitances in the three directions: for example say, 1 micro Farad along the x-direction, $\frac{1}{p}$ micro Farad along the y-direction and $\frac{1}{q}$ micro Farad

along the z-direction. In this case the charge entering the lattice site (l, m, n) is given by:

$$Q_{l,m,n} = 2(1 + p + q)V_{l,m,n} - V_{l+1,m,n} - pV_{l,m+1,n} - pV_{l,m-1,n} - qV_{l,m,n+1} - qV_{l,m,n-1}. \quad (3.11)$$

Therefore the capacitance $C_{l,m,n}$ is still given by Eq. (3.10), but with

$$\alpha = \text{Cos}^{-1}(1 + p + q - p\text{Cos}\beta - q\text{Cos}\gamma). \quad (3.12)$$

Now, with $p = q = 1$ we recover the symmetric SC lattice, while for $p = 1$ and $q = 0$ give the square lattice discussed in section 2 above. Finally, for $p \neq 1$ and $q = 0$ we got the "rectangular" lattice (i.e. a square lattice with unequal capacitances in the two coordinate directions).

4. Results and Discussion

This work is divided into two main parts. In part one, the capacitance between the site $(0, 0)$ and the site (l, m) in an infinite square grid consisting of identical capacitors is calculated using the superposition of charge distribution. The capacitance $C_{l,m}$ is expressed in an integral form which is evaluated analytically and numerically (Table 1). While in part two, the capacitance between the site $(0, 0, 0)$ and the site (l, m, n) in an infinite SC grid consisting of identical capacitors is also calculated using the superposition of charge distribution. The capacitance $C_{l,m,n}$ is expressed in an integral form as the infinite square grid which is evaluated analytically and numerically (Table 2).

In Figs. 1 and 2 the capacitance is plotted against the site (l, m) . Figure 1 shows the capacitance of the infinite square grid as a function of l and m along $[10]$ direction and Fig. 2 shows the capacitance of the infinite square grid as a function of l and m along $[11]$ direction. Inversion symmetry is present in both figures around the origin.

In Figs 3 and 4 the capacitance is plotted against the site (l, m, n) . Figure 3 shows the capacitance of the infinite square grid as a function of l , m , and n along $[100]$ direction and Fig. 4 shows the capacitance of the infinite square grid as a function of l , m , and n along $[111]$ direction. One can notice from these two figures that there are inversion symmetries about the origin.

The asymptotic form of equations (2.15) and (3.9) corresponding to the identical capacitors of infinite square lattice and infinite simple cubic lattice, respectively leads to zero as l goes to infinity (see Figs. 1 and 3).

An investigation of infinite complicated lattices and of lattices with missing capacitor (bond) is in progress.

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Table Captions

Table 1: Numerical values of $C_{l,m}$ in units of C for an infinite square grid.

Table 2: Numerical values of $C_{l,m,n}$ in units of C for an infinite SC grid.

Table 1

l, m	$C_{l,m}$
0	∞
1,0	2
2,0	1.37597
3,0	1.16203
4,0	1.04823
5,0	0.974844
1,1	1.5708
2,1	1.29326
3,1	1.13539
4,1	1.03649
5,1	0.968523
2,2	1.1781
3,2	1.08177
4,2	1.00814
5,2	0.951831
3,3	1.02443
4,3	0.972869
5,3	0.929041
4,4	0.937123
5,4	0.90391
5,5	0.878865

Table 2

l, m, n	$C_{l,m,n}$	l, m, n	$C_{l,m,n}$	l, m, n	$C_{l,m,n}$
0	∞	410	2.144	531	2.08958
100	3.000003	411	2.138018	532	2.084667
110	2.531139	420	2.12867	533	2.077909
111	2.3906	421	2.124356	540	2.080503
200	2.382751	422	2.113825	541	2.079348
210	2.306284	430	2.111192	542	2.075559
211	2.264847	431	2.108277	543	2.070342
220	2.225432	432	2.100721	544	2.064235
221	2.206804	433	2.09079	550	2.070179
222	2.173162	440	2.094776	551	2.069768
300	2.220392	441	2.092865	552	2.066637
310	2.200632	442	2.087565	553	2.062804
311	2.186289	443	2.0803	554	2.057948
320	2.167735	444	2.072238	555	2.05287
321	2.159146	500	2.11299	600	2.088777
322	2.14053	510	2.109767	610	2.087086
330	2.136601	511	2.106833	633	2.069498
331	2.131646	520	2.101692	644	2.056729
332	2.119735	521	2.099336	655	2.047817
333	2.105161	522	2.093075	700	2.071745
400	2.15107	530	2.091324	531	2.08958

Figure Captions

Fig. 1: The capacitance $C_{l,m}$ in terms of l and m for an infinite square grid along the [10] direction.

Fig. 2: The capacitance $C_{l,m}$ in terms of l and m for an infinite square grid along the [11] direction.

Fig. 3: The capacitance $C_{l,m,n}$ in terms of l , m , and n for an infinite SC grid along the [100] direction.

Fig. 4: The capacitance $C_{l,m,n}$ in terms of l , m , and n for an infinite SC grid along the [111] direction.

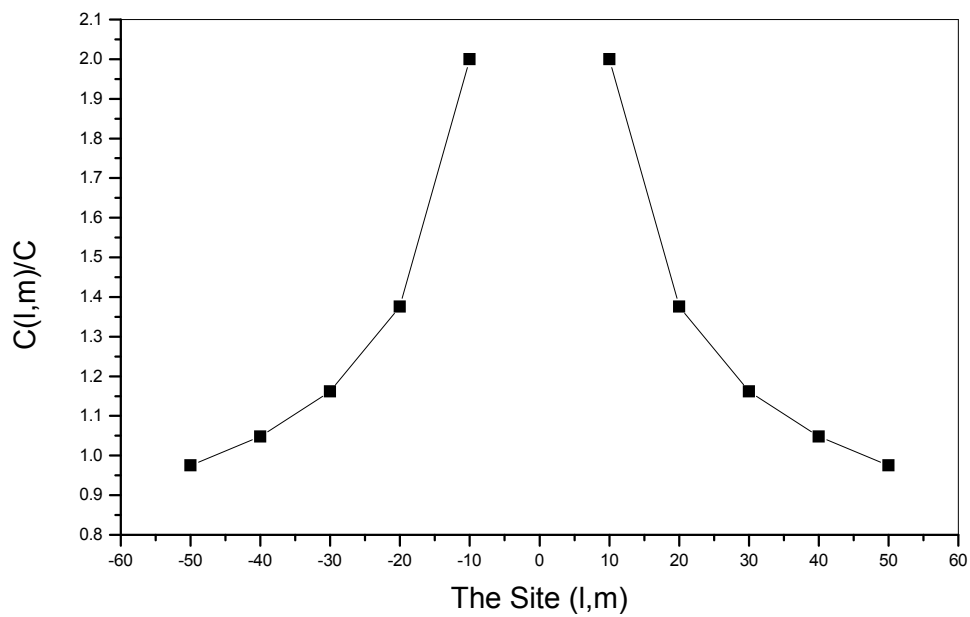


Fig. 1

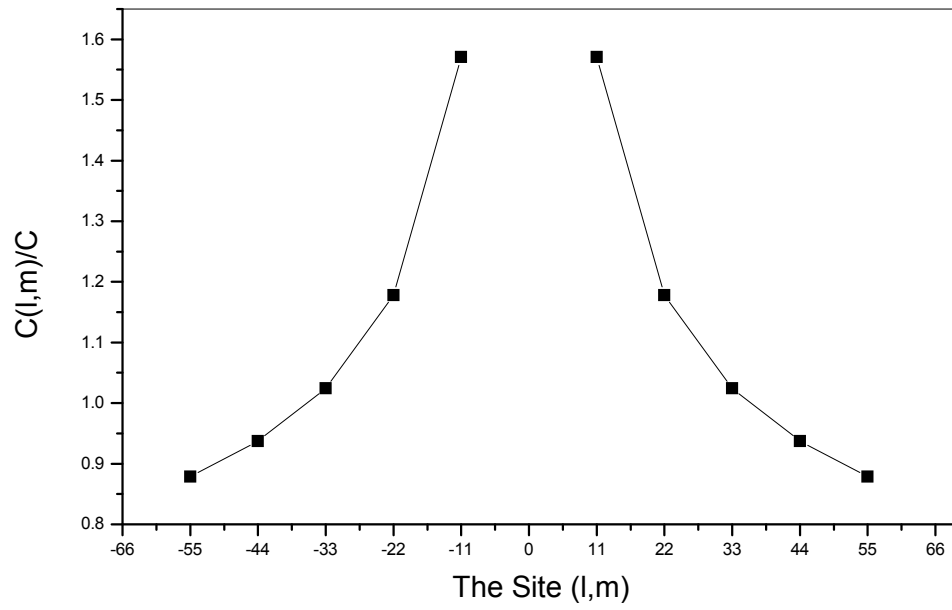


Fig. 2

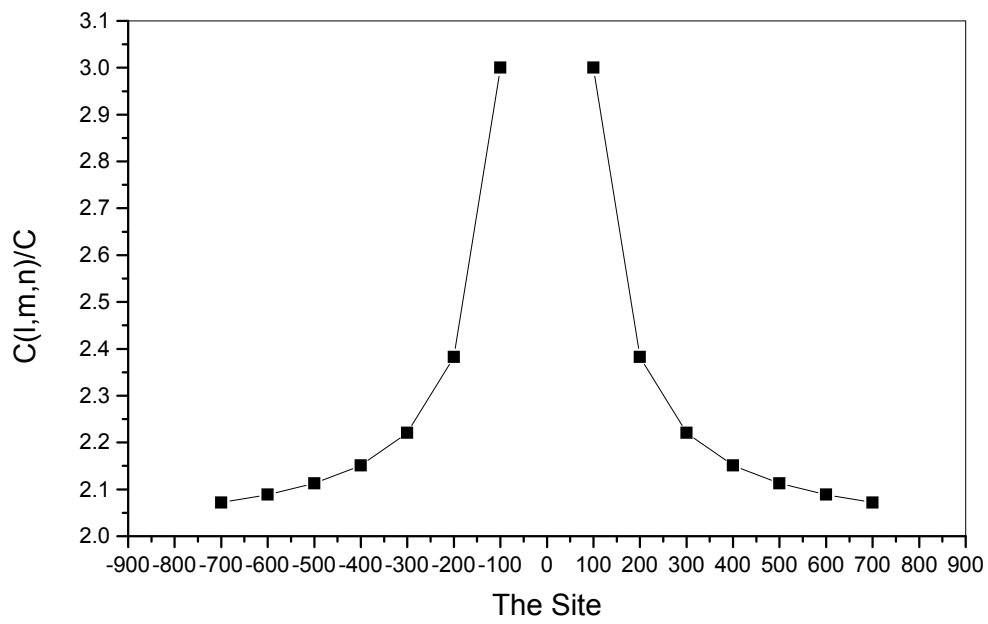


Fig. 3

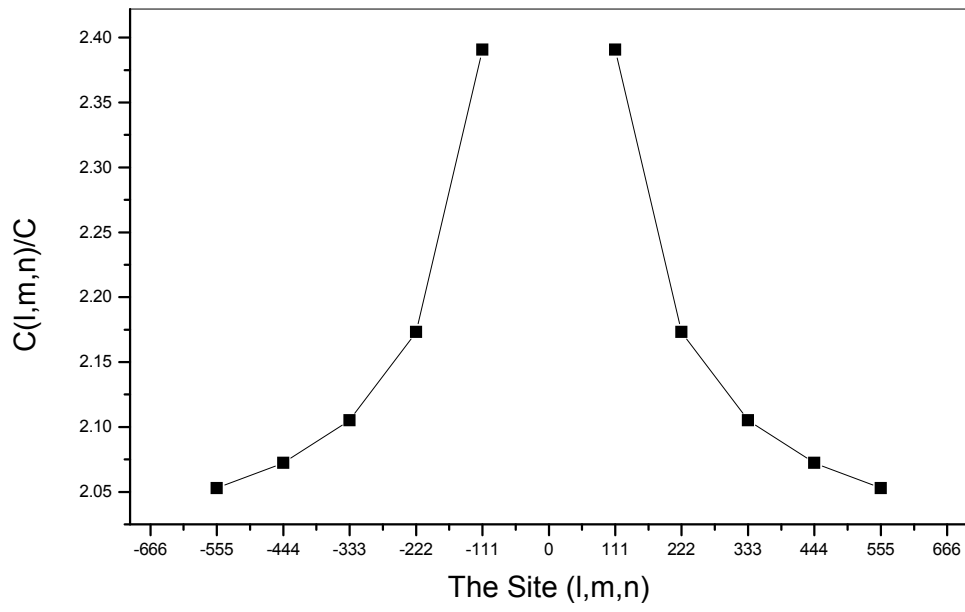


Fig. 4