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A numeric–analytic method for approximating a giving up smoking model containing fractional derivatives

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ABSTRACT

Smoking is one of the main causes of health problems and continues to be one of the world's most significant health challenges. In this paper, the dynamics of a giving up smoking model containing fractional derivatives is studied numerically. The multistep generalized differential transform method (for short MSGDTM) is employed to compute accurate approximate solutions to a giving up smoking model of fractional order. The unique positive solution for the fractional order model is presented. A comparative study between the new algorithm and the classical Runge–Kutta method is presented in the case of integer-order derivatives. The solutions obtained are also presented graphically.

1. Introduction

Tobacco smoking is the leading cause of preventable death, and is estimated to kill more than 5 million people worldwide each year, and this number is expected to grow. According to the World Health Organization report on the global tobacco epidemic [1], tobacco use kills or disables many people in their most productive years, which denies families their primary wage-earners, consumes family budgets, raises the cost of health care and hinders economic development. Smoking or tobacco is a known or probable cause of deaths from cancers of the oral cavity, larynx, lung, oesophagus, bladder, pancreas, renal pelvis, stomach, and cervix. Smoking is also a cause of heart disease, strokes, peripheral vascular diseases, chronic obstructive lung diseases and other respiratory diseases, and low-birth weight babies [2].

Mathematical modelling of complex biological processes is a major challenge for contemporary scientists. In contrast to simple classical biological systems, where the theory of integer-order differential equations is sufficient to describe their dynamics, complex systems are characterized by the variability of structures in them, multiscale behavior and nonlinearity in the mathematical description of the mutual relationship between parameters [3]. Fractional derivatives provide an excellent instrument for the description of the dynamical behavior of various complex biomaterials and systems. The most fundamental characteristic of these models is their nonlocal characteristic which does not exist in the differential operators of integer order. This property means that the next aspect of a model relates not only to its present state but also to all of its historical states. Magin [4] was the first to use fractional derivatives and fractional integrals in order to model the stress–strain relationship in biomaterials. Craiem et al. [5] applied fractional calculus to model arterial viscoelasticity.

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Abdullah [6] used fractional differential equations to model the Michaelis–Menten reaction in a 2-d region containing obstacles. For more details about using fractional calculus in modelling complex biomaterials, we refer the reader to [3] and the references therein.

Some efforts have been made in the mathematical modeling of giving up smoking since the 2000s (see Refs. [7–11]). In [12], the authors proposed a modified model that describes a giving up smoking model. In their paper they studied the quality behavior and represented numerical simulation by using a numerical method. Their model is given by

$$\begin{aligned}
\frac{dP(t)}{dt} &= bN(t) - \beta_1(t)L(t)P(t) - (d_1 + \mu)P(t) + \tau Q(t),\\
\frac{dL(t)}{dt} &= \beta_1(t)L(t)P(t) - \beta_2(t)L(t)S(t) - (d_2 + \mu)L(t),\\
\frac{dS(t)}{dt} &= \beta_2(t)L(t)S(t) - (\gamma + d_3 + \mu)S(t),\\
\frac{dQ(t)}{dt} &= \gamma S(t) - (\tau + d_4 + \mu)Q(t),\\
\frac{dN(t)}{dt} &= (b - \mu)N(t) - (d_1P(t) + d_2L(t) + d_3S(t) + d_4Q(t)),
\end{aligned} \tag{1}$$

under the initial conditions

$$P(0) = c_1, \qquad L(0) = c_2, \qquad S(0) = c_3, \qquad Q(0) = c_4, \qquad N(0) = c_5, \tag{2}$$

where $P(t)$, $L(t)$, $S(t)$, $Q(t)$ and $N(t)$ denote the numbers of potential smokers, occasional smokers, smokers, quit smokers and total smokers at time t, respectively. Here b is the birth rate, μ is the natural death rate, γ is the recovery rate from smoking, $\beta_1(t)$ and $\beta_2(t)$ are transmission coefficients, d_1, d_2, d_3 and d_4 represent the death rates of potential smokers, occasional smokers, smokers and quit smokers related to smoking disease, respectively. Additionally, τ represents the rate at which a quit smoker in the population becomes a potential smoker again.

Now we introduce fractional order into the ordinary differential equation model by Zaman et al. [12]. The new system is described by the following set of fractional order differential equations:

$$D_t^{\alpha}P(t) = bN(t) - \beta_1(t)L(t)P(t) - (d_1 + \mu)P(t) + \tau Q(t), \tag{3}$$

$$D_t^{\alpha}L(t) = \beta_1(t)L(t)P(t) - \beta_2(t)L(t)S(t) - (d_2 + \mu)L(t), \tag{4}$$

$$D_t^{\alpha}S(t) = \beta_2(t)L(t)S(t) - (\gamma + d_3 + \mu)S(t), \tag{5}$$

$$D_t^{\alpha}Q(t) = \gamma S(t) - (\tau + d_4 + \mu)Q(t), \tag{6}$$

$$D_t^{\alpha}N(t) = (b - \mu)N(t) - (d_1P(t) + d_2L(t) + d_3S(t) + d_4Q(t)), \tag{7}$$

where D_t^{α} is a fractional derivative in the Caputo sense and α is a parameter describing the order of the fractional time-derivative with $0 < \alpha < 1$, subject to the same initial conditions given in Eq. (2). The general response expression contains a parameter describing the order of the fractional derivatives that can be varied to obtain various responses. Obviously, the integer-order system can be viewed as a special case of the fractional-order system by putting the time-fractional order of the derivative equal to one. In other words, the ultimate behavior of the fractional system response must converge to the response of the integer order version of the equation.

To the best of our knowledge, this work represents the first available numerical solution for a giving up smoking model of fractional order. For this reason, we intend to obtain the approximate solutions of the problems (3)–(7) via the multi-step generalized differential transform method (MSGDTM). This method is only a simple modification of the generalized differential transform method (GDTM) [13–16], in which it is treated as an algorithm in a sequence of small intervals (i.e. time steps) for finding accurate approximate solutions to the corresponding systems. The approximate solutions obtained by using the GDTM are valid only for a short time. The ones obtained by using the MSGDTM [17,18] are more valid and accurate during a long time, and are in good agreement with the classical Runge–Kutta method numerical solution when the order of the derivative is one.

This paper is organized as follows. In Section 2, we present some necessary definitions and notations related to fractional calculus. In Section 3, we show the existence of the non-negative solution of the giving-up smoking model. In Section 4, the proposed method is applied to the problems (3)–(7) while numerical simulations are presented graphically in Section 5. Finally, the conclusion is given in Section 6.

2. Preliminaries

In this section, we give some basic definitions and properties of the fractional calculus theory which are used further in this paper [19–22].

Definition 1. A function $f(x)$ $(x > 0)$ is said to be in the space C_α $(\alpha \in R)$ if it can be written as $f(x) = x^p f_1(x)$ for some $p > \alpha$ where $f_1(x)$ is continuous in $[0, \infty)$, and it is said to be in the space C_α^m if $f^{(m)} \in C_\alpha$, $m \in \mathbf{N}$.

Definition 2. The Riemann–Liouville integral operator of order $\alpha > 0$ with $a \geq 0$ is defined as

$$(J_a^\alpha f)(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)\, dt, \quad x > a, \tag{8}$$

$$(J_a^0 f)(x) = f(x). \tag{9}$$

Properties of the operator can be found in [19]. We only need here the following:

For $f \in C_\alpha$, $\alpha, \beta > 0$, $a \geq 0$, $c \in R$ and $\gamma > -1$, we have

$$(J_a^\alpha J_a^\beta f)\,(x) = (J_a^\beta J_a^\alpha f)\,(x) = (J_a^{\alpha+\beta} f)\,(x), \tag{10}$$

$$J_a^\alpha x^\gamma = \frac{x^{\gamma+\alpha}}{\Gamma(\alpha)} B_{\frac{x-a}{x}}(\alpha, \gamma+1), \tag{11}$$

where $B_\tau(\alpha, \gamma+1)$ is the incomplete beta function which is defined as

$$B_\tau(\alpha, \gamma+1) = \int_0^\tau t^{\alpha-1}(1-t)^\gamma\, dt, \tag{12}$$

$$J_a^\alpha e^{cx} = e^{ac}(x-a)^\alpha \sum_{k=0}^{\infty} \frac{[c(x-a)]^k}{\Gamma(\alpha+k+1)}. \tag{13}$$

The Riemann–Liouville derivative has certain disadvantages when trying to model real-world phenomena with fractional differential equations. Therefore, we shall introduce a modified fractional differential operator D_a^α proposed by Caputo in his work on the theory of viscoelasticity.

Definition 3. The Caputo fractional derivative of $f(x)$ of order $\alpha > 0$ with $a \geq 0$ is defined as

$$(D_a^\alpha f)(x) = (J_a^{m-\alpha} f^{(m)})\,(x) = \frac{1}{\Gamma(m-\alpha)} \int_a^x \frac{f^{(m)}(t)}{(x-t)^{\alpha+1-m}} dt, \tag{14}$$

for $m-1 < \alpha \leq m$, $m \in \mathbf{N}$, $x \geq a$, $f(x) \in C_{-1}^m$.

The Caputo fractional derivative was investigated by many authors; for $m-1 < \alpha \leq m$, $f(x) \in C_\alpha^m$ and $\alpha \geq -1$, we have

$$(J_a^\alpha D_a^\alpha f)\,(x) = J^m D^m f(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(a) \frac{(x-a)^k}{k!}. \tag{15}$$

For mathematical properties of fractional derivatives and integrals one can consult the mentioned references.

3. Non-negative solutions

Let $R_+^5 = \{X \in R^5 : X \geq 0\}$ and $X(t) = (P(t), L(t), S(t), Q(t), N(t))^T$. For the proof of the theorem about non-negative solutions we shall need the following lemma [23].

Lemma 3.1 (*Generalized Mean Value Theorem*)**.** *Let $f(x) \in C[a, b]$ and $D^\alpha f(x) \in C[a, b]$ for $0 < \alpha \leq 1$. Then we have*

$$f(x) = f(a) + \frac{1}{\Gamma(\alpha)} D^\alpha f(\xi)(x-a)^\xi,$$

with $0 \leq \xi \leq x$, for all $x \in (a, b]$.

Remark 3.2. Suppose $f(x) \in C[0, b]$ and $D^\alpha f(x) \in C[0, b]$ for $0 < \alpha \leq 1$. It is clear from Lemma 3.1 that if $D^\alpha f(x) \geq 0$ for all $x \in (0, b)$, then the function f is non-decreasing, and if $D^\alpha f(x) \leq 0$ for all $x \in (0, b)$, then the function f is non-increasing.

Theorem 3.3. *There is a unique solution for the initial value problem given by (3)–(7), and the solution remains in R_+^5.*

Proof. The existence and uniqueness of the solution of (3)–(7) in $(0,\infty)$ can be obtained from [24, Theorem 3.1 and Remark 3.2]. We need to show that the domain R_+^5 is positively invariant. Since

$$D_t^\alpha P(t)\big|_{P=0} = bN(t) + \tau Q(t) \geq 0,$$
$$D_t^\alpha L(t)\big|_{L=0} = 0,$$
$$D_t^\alpha S(t)\big|_{S=0} = 0,$$
$$D_t^\alpha Q(t)\big|_{Q=0} = \gamma S(t) \geq 0,$$
$$D_t^\alpha N(t)\big|_{N=0} = -(d_1P(t) + d_2L(t) + d_3S(t) + d_4Q(t)) \geq 0,$$

on each hyperplane bounding the non-negative orthant, the vector field points into R_+^5. □

4. MSGDTM method

Although the generalized differential transform method (GDTM) is used to provide approximate solutions for nonlinear problems in terms of convergent series with easily computable components, it has been shown that the approximated solutions obtained are not valid for large t for some systems [13–16]. Therefore we use the multi-step generalized differential transform method (MSGDTM) to approximate the solutions of Eqs. (3)–(7), which offers accurate solution over a longer time frame compared to the standard generalized differential transform method [13,16].

Taking the differential transform of Eqs. (3)–(7) with respect to time t gives

$$\begin{aligned}
P(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}\left(bN(k) - \sum_{k2=0}^{k}\sum_{k1=0}^{k2} B_1(k_1)L(k_2-k_1)P(k-k_2) - (d_1+\mu)P(k) + \tau Q(k)\right),\\
L(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}\left(\sum_{k2=0}^{k}\sum_{k1=0}^{k2} B_1(k_1)L(k_2-k_1)P(k-k_2)\right.\\
&\quad \left. - \sum_{k2=0}^{k}\sum_{k1=0}^{k2} B_2(k_1)L(k_2-k_1)S(k-k_2) - (d_2+\mu)L(k)\right),\\
S(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}\left(\sum_{k2=0}^{k}\sum_{k1=0}^{k2} B_2(k_1)L(k_2-k_1)S(k-k_2) - (\gamma+d_3+\mu)S(k)\right),\\
Q(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}(\gamma S(k) - (\tau+d_4+\mu)Q(k)),\\
N(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}((b-\mu)N(k) - (d_1P(k)+d_2L(k)+d_3S(k)+d_4Q(k))),
\end{aligned} \tag{16}$$

where $P(k)$, $L(k)$, $S(k)$, $Q(k)$, $N(k)$, $B_1(k)$ and $B_2(k)$ are the differential transformations of $P(t)$, $L(t)$, $S(t)$, $Q(t)$, $N(t)$, $\beta_1(t)$ and $\beta_2(t)$. The differential transforms of the initial conditions are given by $P(0) = c_1,\ L(0) = c_2,\ S(0) = c_3,\ Q(0) = c_4$ and $N(0) = c_5$. In view of the differential inverse transforms, the differential transform series solution for the systems (3)–(7) can be obtained as

$$\begin{cases}
P(t) = \sum_{k=0}^{K} P(k)t^{\alpha k},\\
L(t) = \sum_{k=0}^{K} L(k)t^{\alpha k},\\
S(t) = \sum_{k=0}^{K} S(k)t^{\alpha k},\\
Q(t) = \sum_{k=0}^{K} Q(k)t^{\alpha k},\\
N(t) = \sum_{k=0}^{K} N(k)t^{\alpha k}.
\end{cases} \tag{17}$$

Now, according to the multi-step generalized differential transform method, the series solution for the systems (3)–(7) is suggested by

$$P(t)=\begin{cases}\sum_{k=0}^{K}P_1(k)t^{\alpha k}, & t\in[0,t_1]\\ \sum_{k=0}^{K}P_2(k)(t-t_1)^{\alpha k}, & t\in[t_1,t_2]\\ \vdots & \\ \sum_{k=0}^{K}P_M(k)(t-t_{M-1})^{\alpha k}, & t\in[t_{M-1},t_M],\end{cases} \tag{18}$$

$$L(t)=\begin{cases}\sum_{k=0}^{K}L_1(k)t^{\alpha k}, & t\in[0,t_1]\\ \sum_{k=0}^{K}L_2(k)(t-t_1)^{\alpha k}, & t\in[t_1,t_2]\\ \vdots & \\ \sum_{k=0}^{K}L_M(k)(t-t_{M-1})^{\alpha k}, & t\in[t_{M-1},t_M],\end{cases} \tag{19}$$

$$S(t)=\begin{cases}\sum_{k=0}^{K}S_1(k)t^{\alpha k}, & t\in[0,t_1]\\ \sum_{k=0}^{K}S_2(k)(t-t_1)^{\alpha k}, & t\in[t_1,t_2]\\ \vdots & \\ \sum_{k=0}^{K}S_M(k)(t-t_{M-1})^{\alpha k}, & t\in[t_{M-1},t_M],\end{cases} \tag{20}$$

$$Q(t)=\begin{cases}\sum_{k=0}^{K}Q_1(k)t^{\alpha k}, & t\in[0,t_1]\\ \sum_{k=0}^{K}Q_2(k)(t-t_1)^{\alpha k}, & t\in[t_1,t_2]\\ \vdots & \\ \sum_{k=0}^{K}Q_M(k)(t-t_{M-1})^{\alpha k}, & t\in[t_{M-1},t_M],\end{cases} \tag{21}$$

$$N(t)=\begin{cases}\sum_{k=0}^{K}N_1(k)t^{\alpha k}, & t\in[0,t_1]\\ \sum_{k=0}^{K}N_2(k)(t-t_1)^{\alpha k}, & t\in[t_1,t_2]\\ \vdots & \\ \sum_{k=0}^{K}N_M(k)(t-t_{M-1})^{\alpha k}, & t\in[t_{M-1},t_M],\end{cases} \tag{22}$$

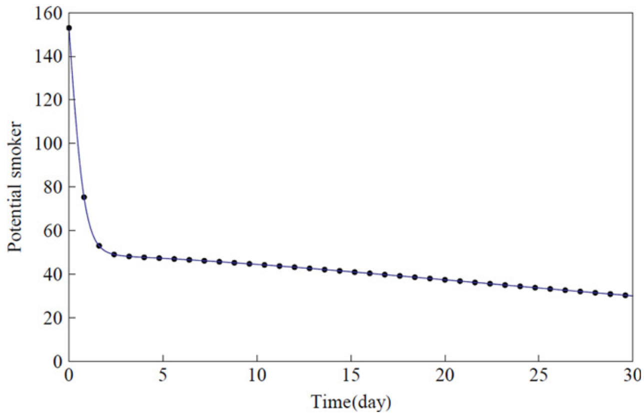

Fig. 1. $P(t)$ versus t: solid line, MSGDTM; dotted line, Runge–Kutta method.

where $P_i(k), L_i(k), S_i(k), Q_i(k)$ and $N_i(k)$ for $i = 1, 2, \dots, M$ satisfy the following recurrence relations:

$$
\begin{aligned}
P_i(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}\left(bN_i(k) - \sum_{k2=0}^{k}\sum_{k1=0}^{k2} B_1(k_1)L_i(k_2-k_1)P_i(k-k_2) - (d_1+\mu)P_i(k) + \tau Q_i(k)\right),\\
L_i(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}\left(\sum_{k2=0}^{k}\sum_{k1=0}^{k2} B_1(k_1)L_i(k_2-k_1)P_i(k-k_2)\right.\\
&\quad \left. - \sum_{k2=0}^{k}\sum_{k1=0}^{k2} B_2(k_1)L_i(k_2-k_1)S_i(k-k_2) - (d_2+\mu)L_i(k)\right),\\
S_i(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}\left(\sum_{k2=0}^{k}\sum_{k1=0}^{k2} B_2(k_1)L_i(k_2-k_1)S_i(k-k_2) - (\gamma+d_3+\mu)S_i(k)\right),\\
Q_i(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}(\gamma S_i(k) - (\tau+d_4+\mu)Q_i(k)),\\
N_i(k+1) &= \frac{\Gamma(\alpha k+1)}{\Gamma(\alpha(k+1)+1)}((b-\mu)N_i(k) - (d_1P_i(k)+d_2L_i(k)+d_3S_i(k)+d_4Q_i(k))),
\end{aligned}
\tag{23}
$$

such that

$$P_i(t_{i-1}) = P_{i-1}(t_{i-1}), \qquad L_i(t_{i-1}) = L_{i-1}(t_{i-1}), \qquad S_i(t_{i-1}) = S_{i-1}(t_{i-1}), \qquad Q_i(t_{i-1}) = Q_{i-1}(t_{i-1})$$

and

$$N_i(t_{i-1}) = N_{i-1}(t_{i-1}), \quad i = 2, 3, \dots, M.$$

Finally, we start with $P_0(0) = c_1$, $L_0(0) = c_2$, $S_0(0) = c_3$, $Q_0(0) = c_4$ and $N_0(0) = c_5$, and, using the recurrence relation given in the system (23), then we can obtain the multi-step generalized differential transform solution given in Eqs. (18)–(22).

5. Numerical results

Eqs. (3)–(7) with transformed initial conditions were solved analytically using the MSGDTM and numerically using the classical Runge–Kutta method in the case of integer-order derivative. To demonstrate the effectiveness of the proposed algorithm as an approximate tool for solving the nonlinear system of fractional differential equations (3)–(7) for larger t, we apply the proposed algorithm on the interval $[0, 30]$. It is to be noted that the multi-step generalized differential transform method results are obtained when $K = 10$ and $M = 3000$. All the results are calculated by using the computer algebra package Mathematica.

We assume the following parameters of system (3)–(7): $b = 0.0045$, $\beta_1(t) = 0.014$, $\beta_2(t) = 0.014$, $\gamma = 0.0165$, $\mu = 0.0021$, $\tau = 0$, $d_1 = 0.034$, $d_2 = 0.045$, $d_3 = 0.054$, $d_4 = 0.061$ and initial conditions $c_1 = 153$, $c_2 = 55$, $c_3 = 79$, $c_4 = 68$, $c_5 = 355$.

Figs. 1–5 show the approximate solutions obtained using the MSGDTM and the classical Runge–Kutta method of $P(t)$, $L(t)$, $S(t)$, $Q(t)$ and $N(t)$ when α is one. From the graphical results in Figs. 1–5, it can be seen that the results obtained using the multi-step generalized differential transform method match the results of the classical Runge–Kutta method very

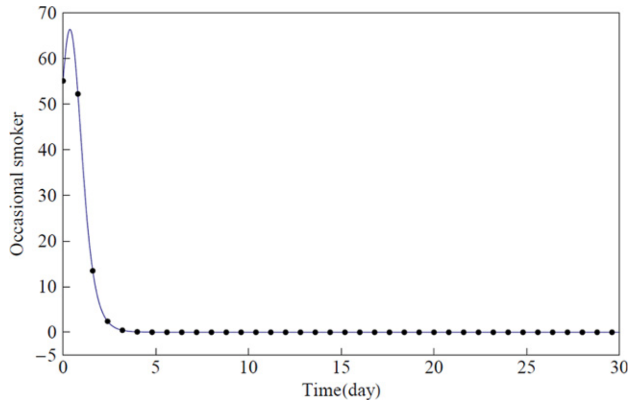

Fig. 2. $L(t)$ versus t: solid line, MSGDTM; dotted line, Runge–Kutta method.

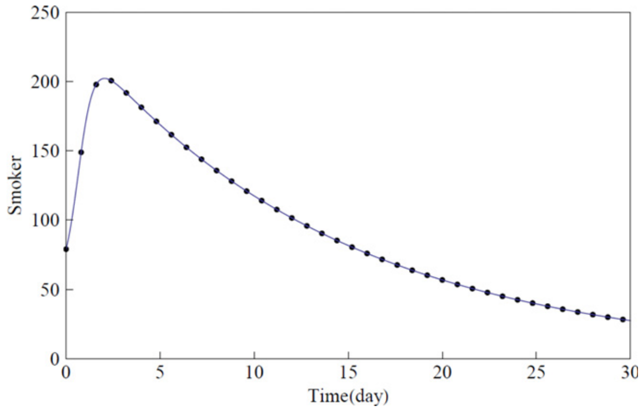

Fig. 3. $S(t)$ versus t: solid line, MSGDTM; dotted line, Runge–Kutta method.

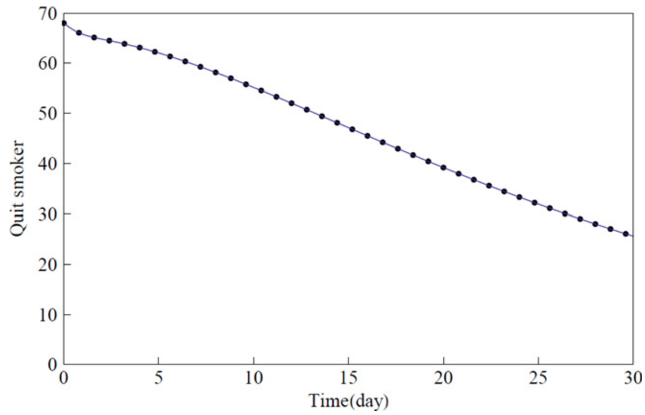

Fig. 4. $Q(t)$ versus t: solid line, MSGDTM; dotted line, Runge–Kutta method.

well, which implies that the presented method can predict the behavior of these variables accurately for the region under consideration.

Figs. 6–10 show the approximate solutions for $P(t)$, $L(t)$, $S(t)$, $Q(t)$ and $N(t)$ obtained for different values of α using the multi-step generalized differential transform method. From the numerical results in Figs. 6–10, it is clear that the approximate solutions depend continuously on the time-fractional derivative α. It is evident that the efficiency of this approach can be dramatically enhanced by decreasing the step size and computing further terms or further components of $P(t)$, $L(t)$, $S(t)$, $Q(t)$ and $N(t)$.

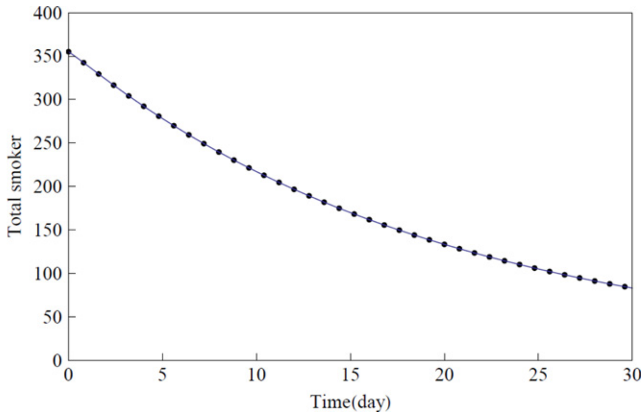

Fig. 5. $N(t)$ versus t: solid line, MSGDTM; dotted line, Runge–Kutta method.

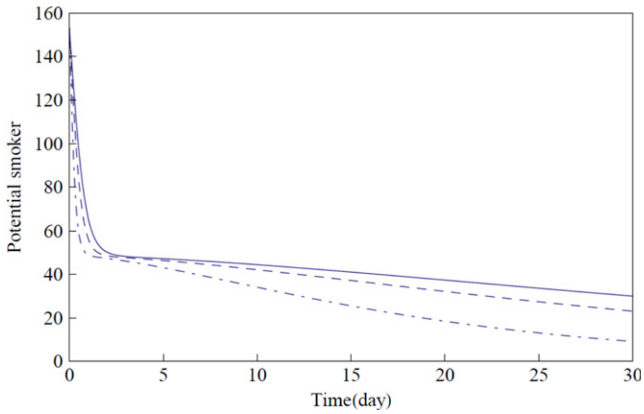

Fig. 6. $P(t)$ versus t: solid line, $\alpha = 1.0$; dashed line, $\alpha = 0.95$; dot-dashed line, $\alpha = 0.85$.

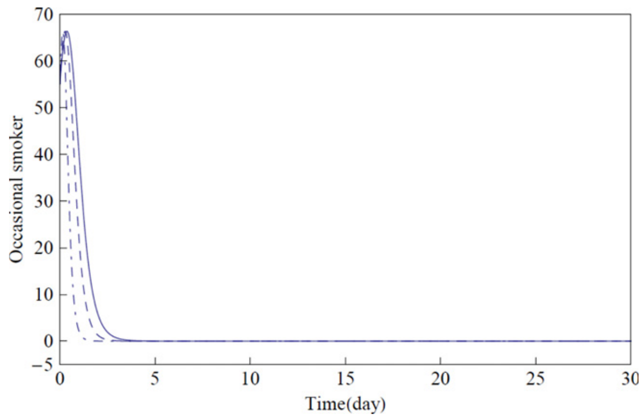

Fig. 7. $L(t)$ versus t: solid line, $\alpha = 1.0$; dashed line, $\alpha = 0.95$; dot-dashed line, $\alpha = 0.85$.

6. Conclusion

In this paper, a fractional order differential system for modeling giving up smoking is studied and its approximate solution is presented using a multi-step generalized differential transform method (MSGDTM). The approximate solutions obtained by the MSGDTM are highly accurate and valid for a long time in the integer case. The reliability of the method and the reduction in the size of the computational domain give this method a wider applicability. Finally, the recent appearance of nonlinear fractional differential equations as models in science and engineering makes it necessary to investigate the method of solution for such equations. Consequently, the present success of the proposed method for the considered model verifes that it is a useful tool for these kinds of models in science and engineering.

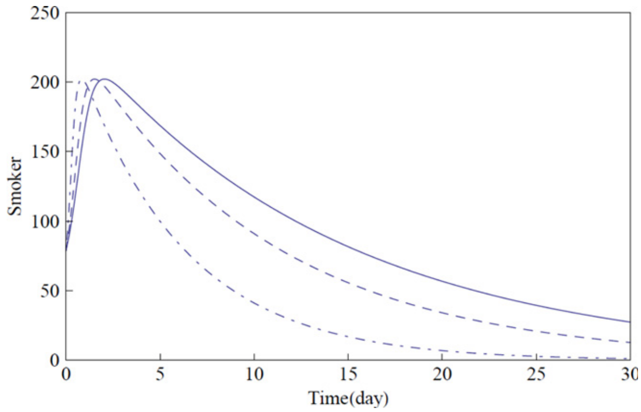

Fig. 8. $S(t)$ versus t: solid line, $\alpha = 1.0$; dashed line, $\alpha = 0.95$; dot-dashed line, $\alpha = 0.85$.

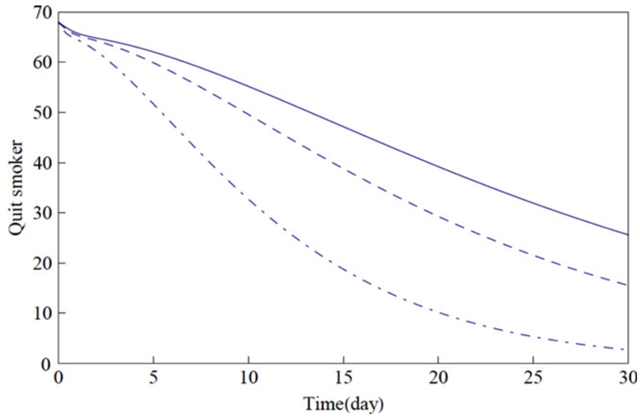

Fig. 9. $Q(t)$ versus t: solid line, $\alpha = 1.0$; dashed line, $\alpha = 0.95$; dot-dashed line, $\alpha = 0.85$.

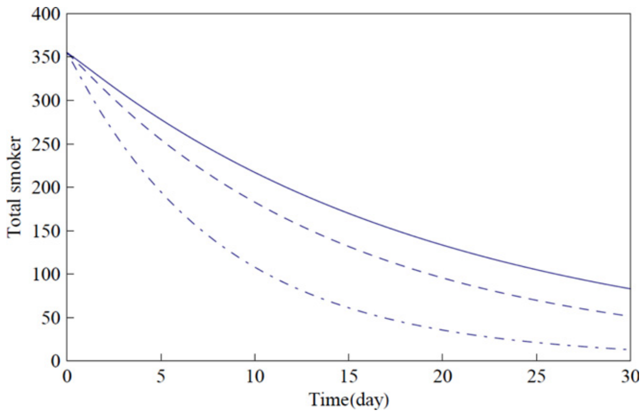

Fig. 10. $N(t)$ versus t: solid line, $\alpha = 1.0$; dashed line, $\alpha = 0.95$; dot-dashed line, $\alpha = 0.85$.

References

[1] World Health Organization report on the global tobacco epidemic, 2009. http://whqlibdoc.who.int/publications/2009/9789241563918_eng_full.pdf.
[2] A. Lahrouz, L. Omari, D. Kiouach, A. Belmaâti, Deterministic and stochastic stability of a mathematical model of smoking, Statistics &Probability Letters 81 (8) (2011) 1276–1284.
[3] J.S. Leszczynski, An Introduction to Fractional Mechanics, Czestochowa University of Technology, Czestochowa, 2011.
[4] R.L. Magin, Fractional Calculus in Bioengineering, Begell House Inc., Redding, 2006.
[5] D.O. Craiem, F.J. Rojo, J.M. Atienza, G.V. Guinea, R.L. Armentano, Fractional calculus applied to model arterial viscoelasticity, Latin American Applied Research 38 (2) (2008) 141–145.
[6] F.A. Abdullah, Using fractional differentialequations to model the Michaelis–Menten reaction in a 2-d region containing obstacles, Science Asia 37 (1) (2011) 75–78.

[7] G. Zaman, Optimal campaign in the smoking dynamics, Computational and Mathematical Methods in Medicine 2011 (2011), Article ID 163834, 9 pages.
[8] G. Zaman, Qualitative behavior of giving up smoking models, Bulletin of the Malaysian Mathematical Sciences Society 34 (2) (2011) 403–415.
[9] J.H. Lubin, N.E. Caporaso, Cigarette smoking, and lung cancer: modeling total exposure and intensity, Cancer Epidemiology, Biomarkers & Prevention 15 (3) (2006) 517–523.
[10] C. Castillo-Garsow, G. Jordan-Salivia, A. Rodriguez Herrera, Mathematical models for the dynamics of tobacco use, recovery and relapse, Technical Report Series BU-1505-M, Cornell University, 2000.
[11] O. Sharomi, A.B. Gumel, Curtailing smoking dynamics: a mathematical modeling approach, Applied Mathematics and Computation 195 (2) (2008) 475–499.
[12] G. Zaman, S. Islam, A non-standard numerical method for a giving-up smoking model, Nonlinear Science Letters A 1 (4) (2010) 397–402.
[13] Z. Odibat, S. Momani, V.S. Ertürk, Generalized differential transform method: application to differential equations of fractional order, Applied Mathematics and Computation 197 (2) (2008) 467–477.
[14] S. Momani, Z. Odibat, A novel method for nonlinear fractional partial differential equations: combination of DTM and generalized Taylor's formula, Journal of Computational and Applied Mathematics 220 (1–2) (2008) 85–95.
[15] Z. Odibat, S. Momani, A generalized differential transform method for linear partial differential equations of fractional order, Applied Mathematics Letters 21 (2) (2008) 194–199.
[16] V.S. Ertürk, S. Momani, Z. Odibat, Application of generalized differential transform method to multi-order fractional differential equations, Communications in Nonlinear Science and Numerical Simulation 13 (8) (2008) 1642–1654.
[17] Z. Odibat, C. Bertelle, M.A. Aziz-Alaoui, G. Duchamp, A multi-step differential transform method and application to non-chaotic or chaotic systems, Computers & Mathematics with Applications 59 (4) (2010) 1462–1472.
[18] V. Erturk, Z. Odibat, S. Momani, An approximate solution of a fractional order differential equation model of human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells, Computers & Mathematics with Applications 62 (3) (2011) 992–1002.
[19] I. Podlubny, Fractional Differential Equations, Academic Press, New York, 1999.
[20] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier, Amsterdam, 2006.
[21] S. Das, Functional Fractional Calculus, Springer, 2011.
[22] R.L. Magin, Fractional Calculus in Bioengineering, Begell House Publishers, 2006.
[23] Z.M. Odibat, N.T. Shawagfeh, Generalized Taylor's formula, Applied Mathematics and Computation 186 (1) (2007) 286–293.
[24] W. Lin, Global existence theory and chaos control of fractional differential equations, Journal of Mathematical Analysis and Applications 332 (1) (2007) 709–726.