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A New Method Of Human Brain Segmentation Utilizing A Class Of Power Series Solutions Of Fractional Differential

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Abstract: A biological dynamic system is very useful in engineering possessions such as control systems, signal processing, and bio-molecular communication networks. On the other hand, fractional differential equations are one of the main subjects that are used to solve problems in biological dynamic systems. In the current paper, we investigate a new class of fractional dynamical systems of brain segmentation. The application shows the improvement of segmentation of a class of brain images.
MSC (2010): 26A33; 34K37

1. Introduction

Fractional operators [1] induced the possessions of memory and heredity of deliveries. Applied difficulties include descriptions of fractional concept allowing the procedure of initial conditions. Fractional time results are related with sub-diffusion of fractional idea, where elements feast more gradually than a traditional diffusion. The fractional operators are operated to model fractional diffusion or distribution where elements buffet at a rate not in arrangement with the traditional Brownian motion model [2]. This operator showed its soundness in all sciences including the medical sciences. The area of medical images has been established and adapted by all fractional operators.

In image processing, the image segmentation is a technique of separating a digital image into multiple pixels. The purpose of this idea is to reduce the illustration of a feature into roughly that is more important and informal to study [3]. Image segmentation is naturally utilized to find bits, pieces and boundaries such as curves, lines, area,… etc in features. Specially, the technique of transmission in a feature such that all pixels remain with fixed segment. Fractional image segmentation is planned by applying different methods from fractional calculus [4-12]. In the current study, we consider the existence of solution of the nonlinear fractional problem with special type by using power series solution. The application is suggested to improve the segmentation of a class of brain images. Comparisons are illustrated with recent studies in fractional segmentation.

2. Fractional processing

Consider the differential equation

$$y^{(\alpha)}(x) = f(x,y), \qquad \alpha \in \Re^{+}, \tag{1}$$

with the initial condition

$$x^{(\alpha-1)}(0) = C \quad (2)$$

where $\Re^+$ is the set of positive real numbers and f is a function from $I \times E$ into E, E being an Euclidean space, and C is a constant. In this section, we shall obtain a solution for the differential equation:

$$y^{(\alpha)}(x) + Cx^{k-\alpha}y(x) = 0 \quad (3)$$

where k is a real constant and $\alpha \in \Re^+$.

First, we required the following definition [1]:

2.1 Definitions

Let f be a function which is defined almost everywhere on [a,b]. For $\alpha > 0$, define the following integral operator

$$\overset{b}{\underset{a}{I}}{}^{\alpha} f = \frac{1}{\Gamma(\alpha)}\int_a^b (b-t)^{\alpha-1} f(t)dt \quad (4)$$

provided that this integral (Lebesgue) exists, where Γ is the Gamma function.

Consequently, the derivative is defined as follows: If $\alpha \in R$, and f is defined on $[a,b]$, we denote $f^{(\alpha)}(x) = \overset{x}{\underset{a}{I}}{}^{-\alpha} f$ for all $x \in (a,b]$, provided that $\overset{x}{\underset{a}{I}}{}^{-\alpha} f$ exists. Fix $\alpha > 0$, and let $n \in N$ be such that $n-1 \le \alpha < n$. From the previous definitions, we have $f^{(\alpha)}(x) = \overset{x}{\underset{a}{I}}{}^{-\alpha} f = D_x^n \overset{x}{\underset{a}{I}}{}^{\alpha-n} f = \frac{d^n}{dx^n}\frac{1}{\Gamma(n-\alpha)}\int_0^x (x-t)^{n-\alpha-1} f(t)dt$ (4). Let $b > -1$. Note that for a power function we have:

$$(x^b)^{\alpha} = \{ \begin{matrix} 0 & \text{if } b \in \{\alpha-1, \alpha-2, ..., \alpha-n\} \\ \Gamma(b+1)/\Gamma(b-\alpha+1)\}x^{b-\alpha} & \textit{otherwise} \end{matrix}.$$

Hence, we shall prove the following proposition:

2.2 Proposition

Let $k \ge n$, and let C be a real constant. Then the equation:

$$y^{(\alpha)}(x) + Cx^{k-\alpha}y(x) = 0 \quad (5)$$

has a solution of the form

$$y(x) = x^{\alpha-n}\sum_{i=0}^{\infty} a_i x^i \quad (6)$$

where

$$a_i = \begin{cases} C & if \quad 0 \le i \le n-1 \\ 0 & if \quad n \le i \le k-1 \end{cases} \quad (7)$$

where C is a constant and

$$a_{i+k} = -Ca_i (i+k-n)!/\Gamma(i+k-n+\alpha+1), \text{ if } i \geq k. \tag{8}$$

Proof: Let $y(x) = x^{\alpha-n}\sum_{i=0}^{\infty} a_i x^i$, where $a_i (i = 0,1,2,...,)$ are define in (7). We consider independently k different subsequences of the sequence $\{a_i\}_{i=0}^{\infty}$. In general, we obtain the sequence $\{a_{j+ki}\}_{i=0}^{\infty}$, $j = 0,1,2,...,k-1$. If $n \leq j < k$, then the subsequence $\{a_{j+ki}\}_{i=0}^{\infty}$ consists of zeros, i.e. $a_{j+ki} = 0$ for all i. Suppose now that $0 \leq j < n$. We have

$$\lim_{i\to\infty} \frac{a_{i+k}}{a_i} = 0, \qquad \lim_{x\to\infty} \frac{\Gamma(x)}{\Gamma(x+\varepsilon)} = 0,$$

for all $\varepsilon > 0$. Therefore, by using the d'Alamberts criteria of convergence of the series, the series is convergent for all $x \in R$. Moreover, it is uniformly convergent in every finite interval $(-r,r), r \in R.$ Thus, we have:

$$y^{(\alpha)}(x) = \frac{d^n}{dx^n}\frac{1}{\Gamma(n-\alpha)}\int_0^x (x-t)^{n-\alpha-1} t^{\alpha-n}\sum_{i=0}^{\infty} a_i t^i dt$$

use the change of variables $t = xs$, we get

$$y^{(\alpha)}(x) = \frac{d^n}{dx^n}\frac{1}{\Gamma(n-\alpha)}\sum_{i=0}^{\infty} a_i x^i \int_0^1 (1-s)^{n-\alpha-1} s^{\alpha-n+i} ds$$

$$= \frac{d^n}{dx^n}\sum_{i=0}^{\infty} a_i x^i \frac{\Gamma(\alpha-n+1+i)}{i!}$$

$$= \sum_{i=n-k}^{\infty} \frac{\Gamma(\alpha+i+k-n+1)}{(i+k-n)!} a_{k+i} x^{i+k-n}$$

$$= \sum_{i=0}^{\infty} \frac{\Gamma(\alpha+i+k-n+1)}{(i+k-n)!} a_{k+i} x^{i+k-n}$$

$$= -Cx^{k-\alpha}\left(x^{\alpha-n}\sum_{i=0}^{\infty} a_i x^i\right) = -Cx^{k-\alpha} y(x)$$

We could do the above calculation (in particular, the change $\left(\int \sum = \sum \int\right)$ because all the series considered above are uniformly convergent on a finite subintervals in the set of real number. This completes the proof.

3. Results and Discussion

In this section, we apply the numerical solutions (6) in a domain. The aim of the technique is to reduce the energy at the outline position in the boundary location where the energy is reduced. The fractional Gaussian kernel (see [4-6]) is utilized in the suggested process to improve the feature texture and retain its construction. Normally, as the usual Gaussian filter is employed, reduce of information sits down as the parameter rises. The fractional Gaussian kernel also increases the arrangement of the inhomogeneous substances in a region. Moreover, it inserts the local feature data within the region to remove the body border throughout the segmentation procedure. In our analysis, we utilize the fractional Gaussian kernel of order $0< \alpha<1$ for two variables (x, y):

$$G_\alpha(x,y)= (1/ (2\pi)^{n/2}\alpha^n) \exp (-(x^2+y^2)/2\alpha^2)$$

The four-sided window (rectangle) changes during the feature to develop the image texture. The window's tool in the planned method does not have specific sizes as it depends on the topology movements in the image. The window effort is basically measured inside the definite domain of the segmented segment, which covers the segment margin angle. The steps of the technique are as follows:

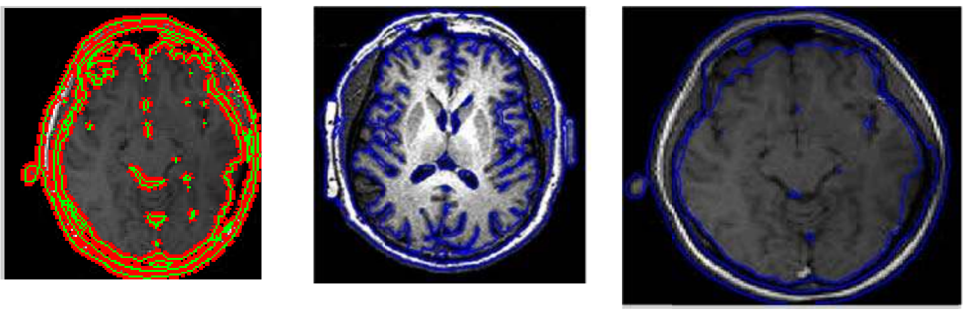
1. Initialization of contour : $I_0 (x,y)$, where $I(x,y)$ is the image;
2. Computing the fractional Gaussian kernel : $G_\alpha(x,y)$ for different values of the fractional power $0< \alpha<1$;
3. Illustrate the window by using the coefficient of the solution in (6);
4. Calculate $a_{i+k} = -Ca_i(i+k-n)!/\Gamma(i+k-n+\alpha+1)$, if $i \geq k$. We shall use C=1.
5. Evaluate the fractional gradient of the image $\Delta^\alpha I(x,y)$ from equation (5);
6. Minimize the energy, by applying the fractional integral equation
$$E_\alpha = \int [G_\alpha(x,y) * \Delta^\alpha I(x,y)]\, dx\, dy$$
7. Measure the contour, by minimizing E_α ; otherwise go to step 2.

The experiment here is considered utilizing Matlab, R (2008b) on a 2.5 GHz Intel Processor i5 giving to the application outline.

For $\alpha= 1$, we get the exact result of the problem (5). Fig. 1 displays that whenever α growth the means absolute error (MAE) reductions and therefore, we obtain the exact outcome at $\alpha= 1$. From above applications, we deduct that the outcomes are approximated to the extended polynomial (6). There are three categories of assessments which are sensitivity, accuracy and specificity [6]. Nevertheless, we focus on the accuracy measure. To manage the assessment, two features of identical medical image are utilized. The first feature is a segmented feature formed by the certain entire active contour model involving ours. The second feature is the medical image with physically tired border. The correctness of the segmentation result is estimated by calculating the space of the physically tired border to the automatic boundary shaped by the active contour model.

4. Conclusions

An investigation has been made of the effects of fractional calculus in segmentation. We considered a new method based of polynomial outcomes of fractional power. The best value of this fractional power is $\alpha=0.88<1$.



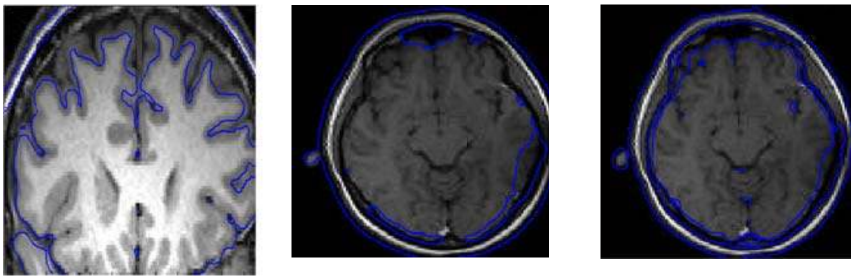


Figure 1. The first row is the experiments and results on images of brain $\alpha=0.88$, $C=1$, $0.1< a_i <1$ by using the proposed method. The second row is the results given in [4], [5] and [6] respectively.

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